GCSE Physics - Problems and Solutions

Vamshi Jandhyala

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Contents

1 Preface

Welcome to this comprehensive collection of physics problems designed to help you excel in your GCSE Physics exam. This book is the result of a collaborative effort and the generosity of the remarkable team at **https://isaacphysics.org/**, who have graciously provided these problems under a **Creative Commons License**. Their dedication to making physics education accessible and engaging has been instrumental in making this book possible.

I want to take a moment to express my heartfelt gratitude to the **isaacphysics.org** team for their outstanding work. Their commitment to creating high-quality educational resources has benefited countless students around the world. I strongly encourage all readers to explore their website **https://isaacphysics.org/** and consider purchasing their excellent physics **http://www.isaacbooks.org/books**, which offer even more valuable content and insights.

As you embark on your journey through these physics problems, remember that practice is key to mastering the subject. Each problem has been carefully selected to cover various aspects of the GCSE Physics curriculum, helping you strengthen your understanding and problem-solving skills. Take your time, work through the problems diligently, and don't hesitate to seek help when needed.

I would also like to extend my deepest appreciation to my family for their unwavering support and cooperation throughout the creation of this book. Their encouragement and understanding have been invaluable, and I am truly grateful for their love and support.

Remember, physics is not just a subject to study, but a fascinating exploration of the world around us. Embrace the challenges, learn from your mistakes, and enjoy the process of discovery. With dedication and perseverance, you will undoubtedly succeed in your GCSE Physics exam and beyond.

Good luck, and may your physics journey be filled with excitement and success!

2 Disclaimer

The problems and solutions in this book have been carefully compiled. However, we cannot guarantee their accuracy, completeness, or suitability for any specific purpose. The author makes no warranties about the information contained herein.

Any reliance on this book's content is at your own risk. The author shall not be liable for any loss or damage arising from the use of or reliance on this book. It is the reader's responsibility to verify the accuracy and applicability of the information provided and use their own judgment when solving problems.

This book is designed to aid in GCSE Physics exam preparation, but success in the exam depends on various factors. This book is a supplementary resource and does not guarantee any specific exam results.

If you have doubts about the accuracy of a problem or solution, consult your teacher, tutor, or official GCSE Physics resources for clarification.

3 Skills

3.1 Units

(i) Complete the table below with the correct SI base units.

Quantity	Equation	Unit in terms of SI base units
Area	$A = L^2$	m^2
Acceleration	$a = \frac{v-u}{t}$	$\frac{m}{s^2}$
Momentum	$p = mv$	kgm/s
Kinetic energy	$E = \frac{1}{2}mv^2$	$kg\frac{m^2}{s^2}$
Gravitational potential energy	$E = mgh$	$kg\frac{m^2}{s^2}$
Electric Charge	$Q = It$	As

(ii) Write the following quantities with the appropriate unit and prefix. Give all answers to 2 significant figures.

0.00000120 m	$1.2\mu m$	5200000 mg	5.2 kg
6500 μs	6.5ms	0.000000920 km	0.92mm
3400000 nA	3.4 mA	0.0000270 kA	27mA
5500000000 nm	5.5 m	6500000 cm^2	650 m^2
0.000044 km/s	4.4 cm/s	83000 mm^3	83 cm^3

(iii) Convert these measurements into metres left bracket (m).

a. 240 cm = 2.4 m

b. 1500 cm = 15 m

c. 95 cm = 0.95 m

d. 7.0 $\times$ 10^3 cm = 70m

(iv) Convert these mass measurements into kilograms (kg).

a. 2500 g = 2.5 kg

b. 350 g = 0.35 kg

c. 1020 g = 1.02 kg

d. 3.80 $\times 10^4$ g= 38 kg

(v) Convert these mass measurements into grams (g).

a. 6.70 kg = 6700 g

b. 3400 mg = 3.4 g

c. 0.050 kg = 50 g

d. 150 mg = 0.15 g

(vi) Convert the following volumes into cubic metres (m^3).

a. 2500 cm^3 = 0.0025 m^3

b. 68 cm^3 = 0.000068 m^3

c. 3700 litres = 3.7 m^3 as 1000 litres is $1m^3$

3.2 Standard Form

(i) Which of the following numbers could be a mantissa?

0.2	**No**
1.2	**Yes**
1.0	**Yes**
10.3	**No**
0.04	**No**
10	**No**
5	**Yes**
7.6	**Yes**

(ii) Express the following numbers in standard form.

4000	4×10^3
0.030	$3 \times 10^{-}2$
8.31	8.31×10^0
0.0000028	$2.8 \times 10^{-}6$

(iii) Write the numbers the normal way.

3×10^3	3000
2×10^{-2}	0.02
76×10^{-3}	0.076
3.54×10^0	3.54

3.3 Rearranging Equations

Rearrange the equations as per the subject variable.

Equation	Subject Variable	Rearranged equation
$F = \frac{kQ_1Q_2}{r^2}$	r	$r = \sqrt{\frac{kQ_1Q_2}{F}}$
$r\left(\frac{2\pi}{T}\right)^2 = \frac{GM}{r^2}$	T	$r = 2\pi\sqrt{\frac{r^3}{GM}}$
$t = \frac{k}{\lambda}$	λ	$\lambda = \frac{k}{t}$
$10(x+y) = 5(x-y)$	x	$x = -3y$
$n = \frac{\sin(i)}{\sin(r)}$	$\sin(r)$	$\sin(r) = \frac{\sin(i)}{n}$
$s = ut + \frac{1}{2}at^2, u = 0$	t	$t = \sqrt{\frac{2s}{a}}$
$E = \frac{1}{2}mv^2$	v	$v = \sqrt{\frac{2e}{m}}$

3.4 Vectors and Scalars

(i) What is the resultant force on a racing car with $24.5kN$ of driving force and $15.2kN$ of opposing frictional forces (i.e. drag)?

Answer

The resultant force is $24.5 - 15.2 = 9.3kN$.

(ii) Why is gravitational field strength a vector quantity?

Answer

Because it has a magnitude and a direction

(iii) What is the resultant magnitude of the displacement if a person walks north $5.00km$ and east $4.00km$? i.e. How far away are they from the starting point?

Answer

The magnitude of the resultant displacement is $\sqrt{5^2 + 4^2} = 6.4km$.

(iv) In what direction does your own weight point when you are not moving?

Answer

Towards the center of the Earth.

(v) In what direction does your own weight point when you are walking sideways?

Answer

Towards the center of the Earth.

(vi) In what direction does your own weight point when you are walking in a circle?

Answer

Towards the center of the Earth.

(vii) A stunt man drives a car out of the back of a moving lorry. For the stunt to work, the car must be moving at a velocity of $-5.00ms^{-1}$ the instant it has left the lorry. The lorry is travelling at a velocity of $25.0ms^{-1}$. What speed must the speedometer on the car reach before the car leaves the lorry?

Answer

The car should have a speed of $30ms^{-1}$.

(viii) Using a scale diagram, calculate the resultant force acting on a sailing boat when an easterly wind provides $2.50kN$ of force, the tide provides $1.20kN$ of force from the direction 30.0° more northerly than the wind. Give your answer to 2 significant figures. Remember that 'an easterly wind' means a wind coming from the East.

Answer

Using Cosine rule, we see that the magnitude of the resultant force is $\sqrt{2500^2 + 1200^2 - 2 \cdot 2500 \cdot 1200 \cdot \cos(150^{\circ})} \approx 3600N$

(ix) A hiker walks $10.0km$ east, $5.00km$ south and $2.00km$ west. Using a scale diagram, calculate his bearing from his start point. Give your answer to the nearest degree.

Answer

The net displacement vector of the hiker makes an angle of $\arctan(\frac{5}{8}) \approx 32^{\circ}$ in the clockwise direction wrt to the x —axis. Therefore the bearing from his starting point is $90^{\circ} + 32^{\circ} = 122^{\circ}$.

(x) A kite is in equilibrium, so the total sum of the forces is equal to zero. On a vector diagram, the arrows representing the forces would form a closed loop. Three forces act on the kite; the force from the wind, the weight of the kite and the tension in the string. The wind produces a horizontal force of 70.0N and an upward force of 50.0N and the kite weighs 25.0N.

Answer

The net upward force is $50 - 25 = 25N$. The net horizontal force is $70N$. The total force due to the wind and the weight of the kite has to be balanced by the tension in the string. The tension in the string is $\sqrt{25^2 + 70^2} = 74.33N$.

The acute angle the string makes to the horizontal is given by $\arctan(\frac{25}{70}) = 20^{\circ}$.

3.5 Variables and Constants

(i) Scientists wish to know the acceleration of a car as it rolls down a sloping ramp. They set the ramp at a certain angle and then release the car from different positions up the ramp, timing how long it takes to reach the bottom. There are several quantities that can be changed in this experiment.

For each of the following, state whether it is a control variable, independent variable or dependent variable.

Answer

Variable	**Variable Type**
Length of the ramp	Control
Distance the car rolls	Independent
Duration of the car's motion	Dependent
Mass of the car	Control
Angle of the ramp	Control
Surface material of the ramp	Control

The independent variable (distance) is changed during the experiment, and we observe changes in the dependent variable (duration of motion). Everything else is kept constant.

(ii) A sportsman wants to know the bouncing efficiency of a table tennis ball. He drops the ball from various heights and measures the maximum height the ball reaches after the first bounce. For each of the quantities listed in the table, state whether it is a control variable, an independent variable or a dependent variable.

Answer

Variable	**Variable Type**
Size of the ball	Control
Material of the ball	Control
Height of ball before being dropped	Independent
Maximum height of the ball after one bounce	Dependent
Mass of the ball	Control

Material of surface onto which the ball is dropped	Control

The independent variable (height before being dropped) is changed during the experiment, and we observe changes in the dependent variable (maximum height after one bounce). Everything else is kept constant.

3.6 Straight Line Graphs

(i) A student wishes to measure the resistance, R, of a fixed resistor by varying the potential difference, V, across it and measuring the current, I, that flows through it. These quantities are related by $V = IR$. You might find it useful to re-write this relation as $I = \left(\frac{1}{R}\right) \times V$. The student plots V on the x axis.

What variable should be plotted on the y-axis?

Answer

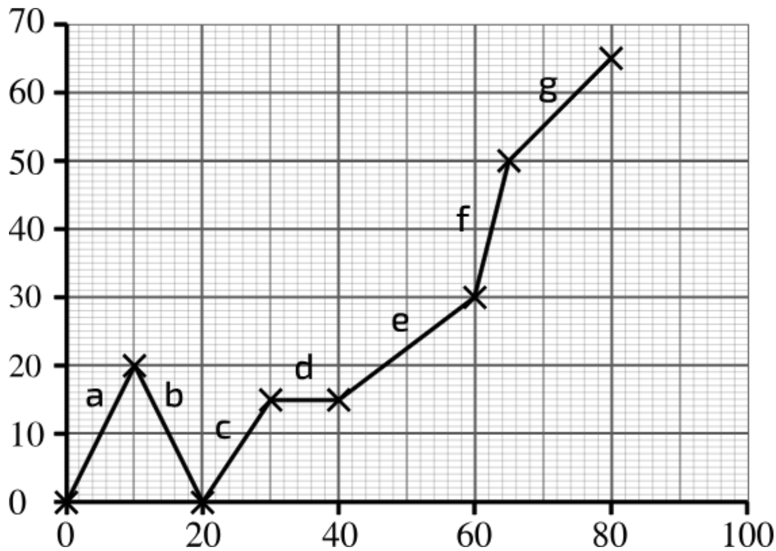
Current

How can the resistance of the fixed resistor be determined from the graph?

Answer

$\left(\frac{1}{gradient}\right)$ equals the resistance.

(ii) Calculate the gradient of the lines labelled a, b, c, d, e, f and g.



Answer

(i) The gradient of a is $\frac{10-0}{20-0} = 2.$

(ii) The gradient of b is $\frac{0-20}{20-10} = -2.$

(iii) The gradient of c is $\frac{15-0}{30-20} = -\frac{1}{2}.$

(iv) The gradient of d is $0.$

(v) The gradient of c is $\frac{30-15}{60-40} = 0.75.$

(vi) The gradient of c is $\frac{50-30}{65-60} = 4.$

(vii) The gradient of c is $\frac{65-50}{80-65} = 1.$

(iii) Write the equation of a line which has a gradient of 2, if $y = 5$ when $x = 0$.

Answer

The equation of the line is $y - 5 = 2(x - 0) \Rightarrow y = 2x + 5$.

(iv) Write the equation of a line with gradient of 5, if $y = 7$ when $x = 1$.

Answer

The equation of the line is $y - 7 = 5(x - 1) \Rightarrow y = 5x + 2$.

(v) Write the equation of a line with gradient of -8, if $y = 0$ when $x = 5$.

Answer

The equation of the line is $y - 0 = -8(x - 5) \Rightarrow y = -8x + 40$.

3.7 Proportionality

(i) The braking force required to stop a car is inversely proportional to the time taken to stop it. If a $5500N$ force can stop the car in $8.0s$, how much force would be needed to stop it in $3.5s$?

Answer

If F is the braking force, we have the equation $F = \frac{k}{t}$ where k is the constant of proportionality. Therefore,
$5500 \times 8 = F \times 3.5 \Rightarrow F = 12571N$

(ii) The current through a resistor is inversely proportional to its resistance. With a 330Ω resistor, the current is $25mA$. What value of resistance is needed if you wish a $55mA$ current to pass?

Answer

If I is the current, we have the equation $I = \frac{k}{R}$ where k is the constant of proportionality.. Therefore, $25 \times 330 = 55 \times R \Rightarrow R = 150\Omega.$

(iii) Which two criteria must be met for a line graph to indicate direct proportionality between two quantities?

Answer

It is a straight line through the origin.

(iv) For each of the following equations state whether the two stated variables are directly proportional, inversely proportional or neither. [If there are other values in the question, they are kept constant.]

  (i) Are W and m directly proportional, inversely proportional or neither in the equation $W = mg$?

  Answer

  Directly proportional.

  (ii) Are p and V directly proportional, inversely proportional or neither in the equation pV=kT?

  Answer

  Inversely proportional.

  (iii) Are p and v directly proportional, inversely proportional or neither in the equation $p = mv$?

  Answer

Directly proportional.

(iv) Are F and r directly proportional, inversely proportional or neither in the equation $F = k\frac{Q_1 Q_2}{r^2}$?

Answer

Neither.

(v) Are $T(K)$ and $T(\circ C)$ directly proportional, inversely proportional or neither in the equation $T(K) = T(\circ C) + 273$?

Answer

Neither.

(vi) Are a and f^2 directly proportional, inversely proportional or neither in the equation $a = 4\pi^2 r f^2$?

Answer

Directly proportional.

4 Mechanics

4.1 Speed, Distance and Time

(i) Use the formula speed = distance/time to calculate the missing values from the table.

Distance(m)	**Time(s)**	**Speed(m/s)**
100	10	**10**
990	3	**330**
2.0×10^3	5	**400**
3300	10	330m/s
15×10^8	5	3.0×10^8
3600	**300**	12
1.2×10^9	4	3.0×10^8
1.7×10^4	50	340

(ii) Work out the missing measurements from the following table, where each row is a separate question.

Average speed	Total Distance	Total time
330m/s	**3300m**	10.0s
300000 km/s	6.00 km	20 μs
3.0 m/s	45 m	**15 s**
1.67 km/h	40.1 km	24 hrs
29.8km/s	940 Gm	$32 \times 10^6 km/h$
0.0470km/h	**0.002 km**	2min 33s
12m/s	100 m	8.13s

(iii) A train has an average speed of 100 km/h. Explain why the maximum speed could be different.

Answer

We get the average speed by dividing the total distance by the total time. The **instantaneous** speed can be either less than or greater than the average speed at any given point in time. E.g.there could be sections where the train must go slower than maximum speed, e.g through stations, so the average speed is different to the maximum.

(iv) How far can you run in 15 seconds at an average speed of 8.0m/s?

Answer

$\textbf{Total Distance} = \textbf{Average Speed} \times \textbf{Total time}$. Therefore, the total distance is $8m/s \times 15s = 120m$.

(v) How long does a car take to travel 2.4km at an average speed of 30m/s?

Answer

$\textbf{Total time} = \textbf{Total Distance / Average Speed}$. We see that $2.4km = 2400m$. Therefore, the total time is $\frac{2400m}{30m/s} = 80s$.

(vi) A good long distance runner has an average speed of 5.5m/s. How far would the runner go in 30 minutes?

Answer

$\textbf{Total Distance} = \textbf{Average Speed} \times \textbf{Total time}$. We see that $30minutes = 30minutes \times 60seconds/minute = 1800seconds$. Therefore, the total distance is $5.5m/s \times 1800s = 9900m = 9.9km$.

(vii) The London-Glasgow shuttle takes approximately 60 minutes to fly a distance of 650km. Calculate its average speed in m/s.

Answer

$\textbf{Average Speed} = \textbf{Total Distance / Total time}$. We see that $60mins = 3600s$ and $650km = 650,000m$. Therefore, the average speed of the shuttle is $\frac{650000}{3600} = 180.56m/s$.

(viii) The wandering albatross can fly at speeds of up to 32m/s (the speed limit on motorways!). One albatross was found to have flown 16250km in 10days.Calculate its average speed in metres per second.

Answer

$\textbf{Average Speed} = \textbf{Total Distance / Total time}$. We see that $10days = 10days \times 24hrs/day \times 3600s/hr = 864000s$ and $16250km = 16250,000m$. Therefore, the average speed of the albatross is $\frac{16250000}{864000} = 18.80m/s$.

(ix) A cross-channel ferry travels at about 7m/s. At the same average speed, how long would it take to cross the Atlantic Ocean, a distance of 6700km? Answer in hours to the nearest hour.

Answer

$\textbf{Total time} = \textbf{Total Distance / Average Speed.}$ We see that $\mathbf{6700}km = \mathbf{6700,000}m.$ Therefore, the total time is $\frac{6700000m}
{7m/s}= 957143s = \frac{957143}{3600}hrs \thickapprox 266hrs$.

(x) A 'light-year' is the distance travelled through space by light in a year. Light travels 300 million metres every second. How far is one light-year in kilometres? Answer to 2sf.

Answer

4.2 Displacement and Distance

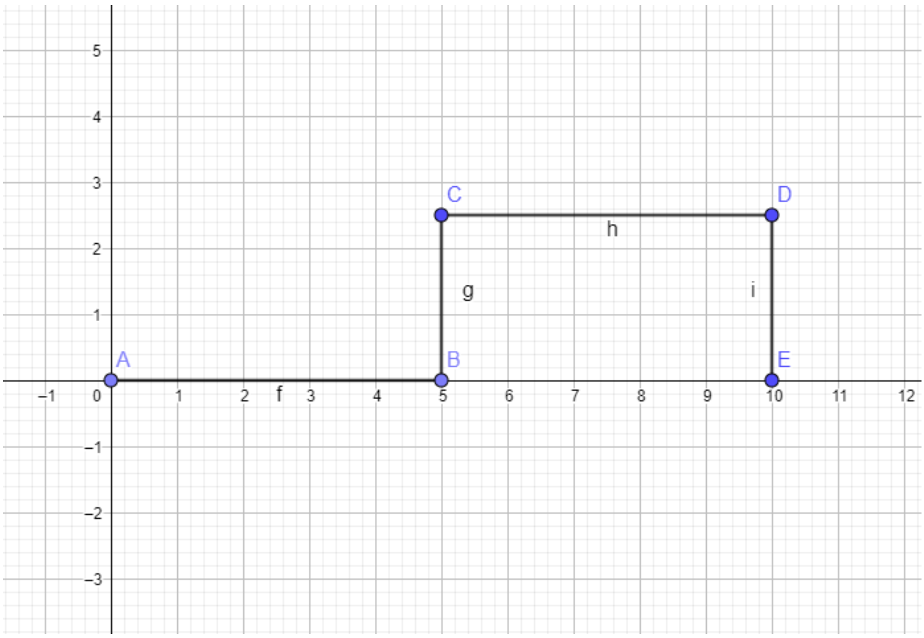
(i) a. A bus travels 500m east, 250m north, 500m east and 250m south. What distance has the bus travelled?

Answer

The distance travelled by the bus = 500m + 250m + 500m + 250m = 1500 m = 1.5km.

b. What is the length of the final displacement of the bus?

Answer



The length of the final displacement is given by the length of AE in the above figure which is $1000m = 1km$.

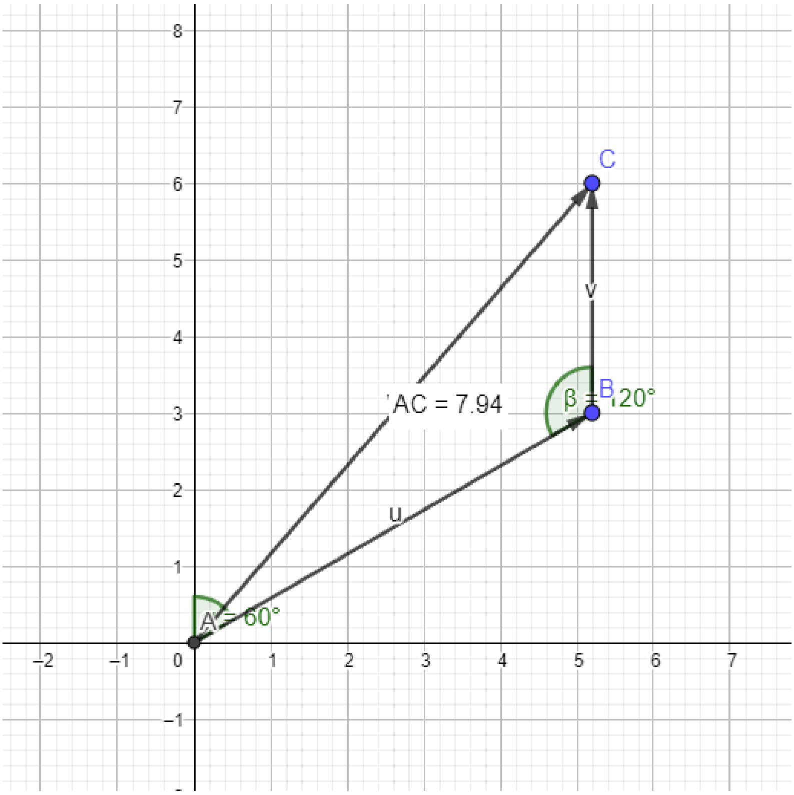
(ii) A climber climbs 50.0m up a vertical cliff face, before being forced to climb back down 5.00m so that she can find an alternative route to the top of the cliff. She climbs sideways to the left 10.0m, then continues to climb 55.0m to the top.

Answer

The distance travelled by the climber = 50m + 5m + 10m + 55m = 120m.

(iii) An object moves 60.0m on a bearing (angle from North) of 60.0°. If the object then moves 30.0m North, how far is it from the start point? You may use a scale diagram or trigonometry to answer this question.

Answer



The required distance is given by length of $\overrightarrow{AC}$. The length of $\overrightarrow{AB}$ = 60m and length of $\overrightarrow{BC}$ = 30m. Using cosine rule, we see that the length of $\overrightarrow{AC}$ is

$$\sqrt{60^2+30^2-2\cdot 60\cdot 30\cos 120^{\circ}} = \sqrt{60^2+30^2+60\cdot 30\sqrt{3}} = 79.41(m)$$

(iv) a. A box is dropped from an aeroplane 2000m high travelling horizontally at 100m/s. The box takes 20.2s to hit the ground. While the box speeds up vertically, it continues at 100m/s horizontally.What distance has the box travelled horizontally when it hits the ground?

Answer

The **horizontal** distance travelled by the box before it hits the ground is $100m \times 20.2s = 2020m$. The key assumption here is that there is no force acting on the box in the horizontal direction. Hence, there is no acceleration in the horizontal direction.

b. When the box hits the ground, what is the magnitude of the displacement of the box from the location it was released? i.e. how far is the box from the release point?

Answer

To calculate the magnitude of the displacement of the box from the location it was released, we need the magnitude of the horizontal as well as the vertical displacement. The magnitude of the horizontal displacement is $2020m$ as calculated in the previous section. The magnitude of the vertical displacement of the box if $2000m$. Therefore the magnitude of the total displacement is given by $$\sqrt{2000^2 + 2020^2 } = 2842.6m \thickapprox 2.8\times10^3 m.$$

c. What is the angle between the horizontal and the displacement of the box from the location it was released? Give your answer to 3 significant figures.

Answer

The angle between the horizontal and the displacement of the box from the location it was released is given by $$\arctan{\frac{2000}{2020}} \thickapprox 44.7^{\circ }.$$

(v) A bridge is a quarter of a circle of radius 20.0m.

a. What distance does a car travel whilst crossing the bridge?

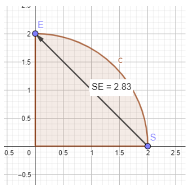
Answer

The distance travelled by the car is given by the length of the quarter circle (quarter of the circumference) of radius 20m = $\frac{2 \pi 20m}{4} \thickapprox 31m$.

b. What is the magnitude of the displacement of the car from the start of the bridge to when it leaves the bridge?

Answer

In the figure below S is the starting point of the bridge and E is the point at which the car leaves the bridge.



The magnitude of the displacement of the car from the start of the bridge to when it leaves the bridge is given by length of $\vec{SE} \thickapprox 28m$.

(vi) A car is moving at a speed of 10.0m/s. It is on a roundabout with a diameter of 50.0m. After 23.56s on the roundabout:

a. What distance has the car travelled?

Answer

The total distance travelled by the car in $23.56s$ with a constant speed of $10m/s$ is $\thickapprox 236 m$.

b. How many turns of the roundabout has the car made?

Answer

When the car travels a distance equal to the circumference of circle with radius $25m$, it completes one turn of the roundabout. Therefore the number of turns of the roundabout is $\frac{236 m}{2 \pi 25 m} \thickapprox 1.5$ turns.

c. What is the magnitude of the car's displacement from where it entered the roundabout?

Answer

The magnitude of the car's displacement from the point of entry to the point after which it completes 1.5 turns is equal to the diameter of the roundabout which is 50m.

d. Other than at the very start, t=0s, is it possible for the distance the car has travelled to equal its displacement at any point on the roundabout?

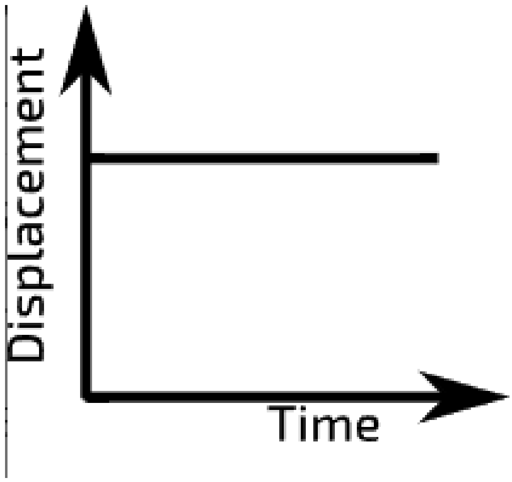
Answer

No, as the distance travelled is an arc of the circumference and the displacement is a chord of the roundabout and these cannot be equal.

4.3 Motion Grapghs: Displacement-Time

(i) Describe the motions of the object for which the following displacement-time graphs have been constructed.

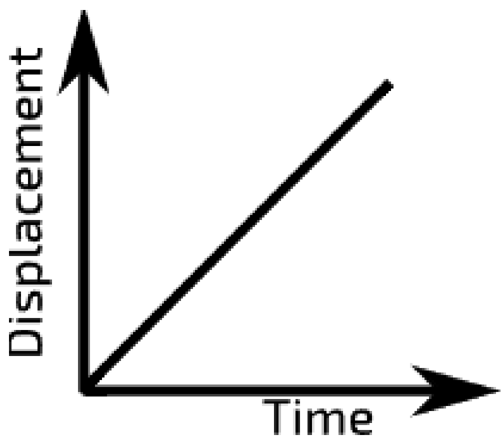
a. No gradient



Answer

As the displacement is not changing with time, the body is not moving.

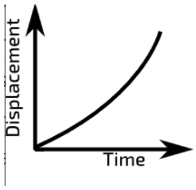
b. Constant gradient



Answer

As the displacement-time graph has a constant positive gradient, the body is moving with a positive constant velocity.

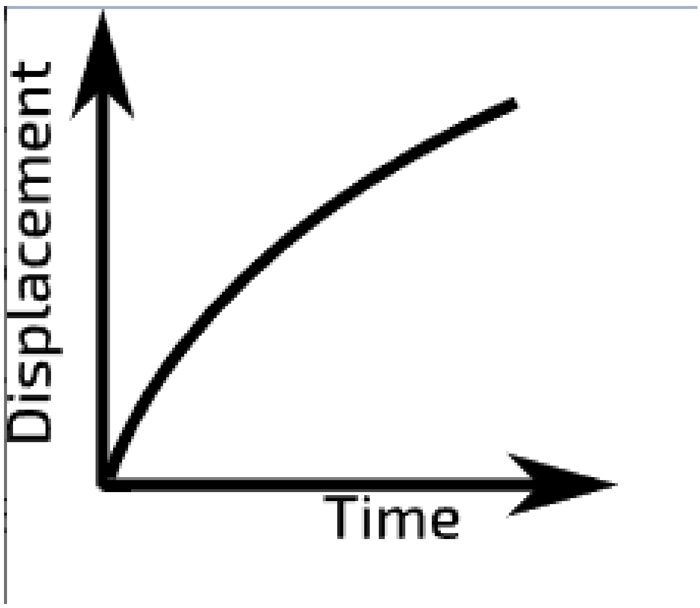
c. Increasing gradient



Answer

As the displacement-time graph has an increasing gradient, the body is accelerating.

d. Decreasing gradient



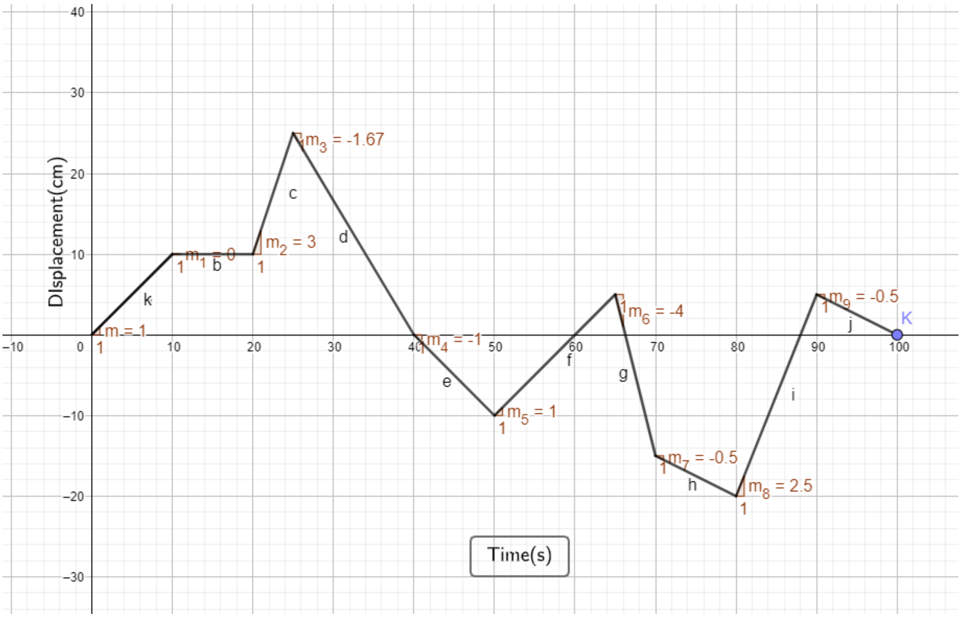
Answer

As the displacement-time graph has an increasing gradient, the body is decelerating.

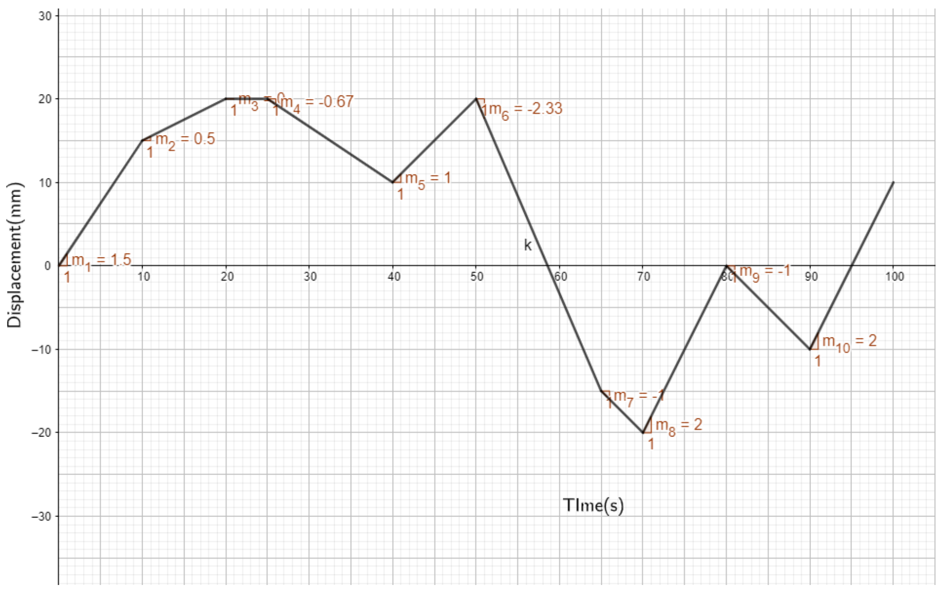
(ii) For the graph below, calculate the velocity for each labeled section a-j.

Answer

The velocity for each labelled section is given by the gradients in the following figure.



(iii) Considering the graph below:



(i) Between which times is the velocity most negative? There may be more than one correct answer.

Answer

50s to 65s

(ii) Calculate the most negative velocity during the motion.

Answer

−2.3 mm/s

(iii) Between which of these times is the velocity most positive?

Answer

70 - 80 seconds

(iv) Calculate the most positive velocity during the motion.

Answer

2 m/s

(v) Between which times is the speed highest?

Answer

50 to 65 s

(vi) What is the highest speed during the motion.

Answer

2.3 mm/s

(vii) Between which times is the speed lowest?

Answer

20s to25s

(viii) Calculate the lowest speed during the motion (to the nearest integer).

Answer

0 mm/s

4.4 Acceleration

(i) A tennis ball is thrown in the air upwards at 15m/s. If it is accelerating downwards at $10m/s^2$, what will its velocity be 2.0s after it is thrown? Let downwards vectors be positive and upwards negative.

Answer

Final velocity v is related to the initial velocity u and acceleration a using the formula $v = u + at$.

We have $u = -15m/s$ and $a = 10m/s^2$. Therefore, v at $t = 2s$ is given by

$$v = -15 + 2 \cdot 10 = 5m/s. \tag{4.1}$$

(ii) A rollercoaster speeds up from rest to 100mph(45m/s) in 1.2s. Calculate the acceleration.

Answer

We have $u = 0m/s$ and $v = 45m/s$. Therefore the acceleration is

$$a = \frac{v-u}{t} \implies a = \frac{45-0}{1.2} = 37.5m/s^2. \tag{4.2}$$

(iii) A car starts from rest and reaches a speed of 40m/s in a time of 8.0 seconds. Calculate its average acceleration.

Answer

The average acceleration is $\frac{40-0}{8} = 5m/s^2$.

(iv) A certain make of car can reach 60mph from rest in a time of 9.0 seconds. Calculate its average acceleration in m/s^2.[Note: 1mph=0.45m/s].

Answer

The average acceleration is $\frac{60\times0.45-0}{9} = 3m/s^2$.

(v) Calculate the change of speed of a train which accelerates for 9.0seconds at a rate of 1.2m/s^2 in a straight line.

Answer

The change in speed is $9 \times 1.2 = 10.8m/s$.

(vi) In overtaking a lorry on a straight section of road, a driver increases speed from 50mph to 70mph in 5.0s. [Note: 1mph=0.45m/s]. Calculate the acceleration in:

a. Miles per hour per second

Answer

The acceleration is $\frac{70-50}{5} = 4mph/s$.

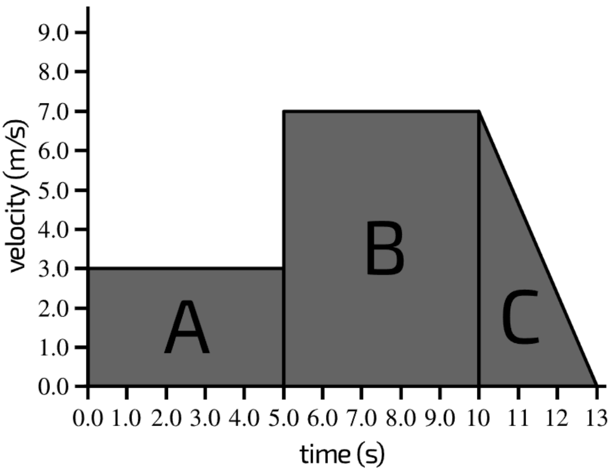
b. Metres per second per second

Answer

The acceleration is $\frac{0.45(70-50)}{5} = 1.8m/s^2$.

4.5 Motion Graphs: Velocity-Time

(i) Using the following speed-time graph:



a. Calculate the distance travelled in A.

Answer

The distance travelled in A is the area of rectangle $\boldsymbol{A = 3 \times 5 = 15m.}$

b. Calculate the distance travelled in B.

Answer

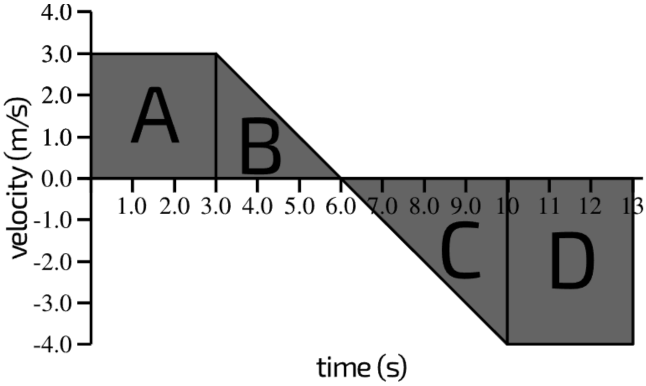
The distance travelled in B is the area of rectangle $\boldsymbol{B = 7 \times (10 - 5) = 35m.}$

c. Calculate the distance travelled in C.

Answer

The distance travelled in C is the area of triangle $\boldsymbol{C = \frac{1}{2} \times 7 \times (13 - 10) = 10.5m.}$

(ii) Look at this graph to answer the questions below.



a. Calculate the displacement in A.

Answer

The displacement in A is area of the square $A = 3 \times 3 = 9m.$

b. Calculate the displacement in B.

Answer

The displacement in B is the area of triangle $B = \frac{1}{2} \times 3 \times (6-3) = 4.5m.$

c. Calculate the displacement in C.

Answer

The displacement in C is the area of triangle $C = \frac{1}{2} \times (-4) \times (10-6) = -8m.$

d. Calculate the displacement in D.

Answer

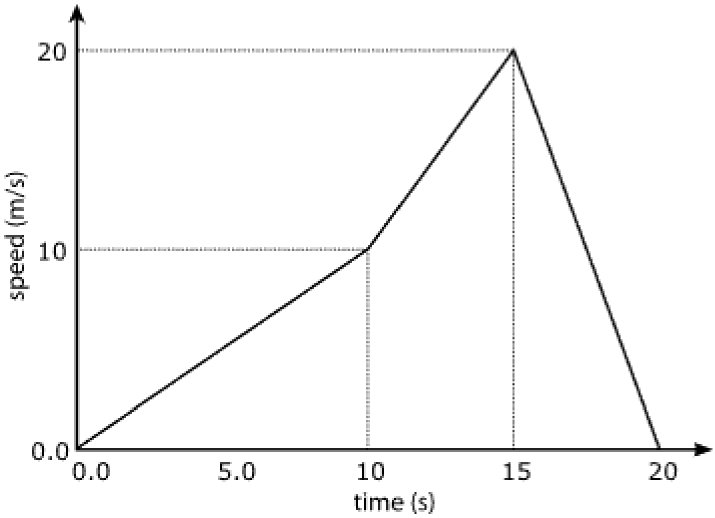
The displacement in D is the area of rectangle $D = (-4) \times (13-10) = -12m.$

e. Calculate the total displacement.

Answer

The total displacement is $9 + 4.5 - 8 - 12 = -6.5m.$

(iii) For the motion described by the following speed-time graph, calculate:



a. The distance moved in the first 10s.

Answer

The distance moved in the first 10s is given by the area of the triangle $\frac{1}{2} \times 10 \times 10 = 50m$.

b. The distance moved in the first 15s.

Answer

The distance moved in the first 15s is given by the area of the triangle + area of the trapezium = 50 + $\frac{1}{2} \times (10 + 20)(15 - 10) = 125m$.

c. The total distance moved

Answer

The total distance moved is given by the area of the triangle + area of the trapezium + area of the second triangle = 125 + $\frac{1}{2} \times (20)(20 - 15) = 175m$.

d. The acceleration in the first 10s.

Answer

The acceleration in the first 10s is given by the gradient of the line segment = $\frac{10-0}{10-0} = 1m/s^2$.

e. The acceleration between 10 to 15s.

Answer

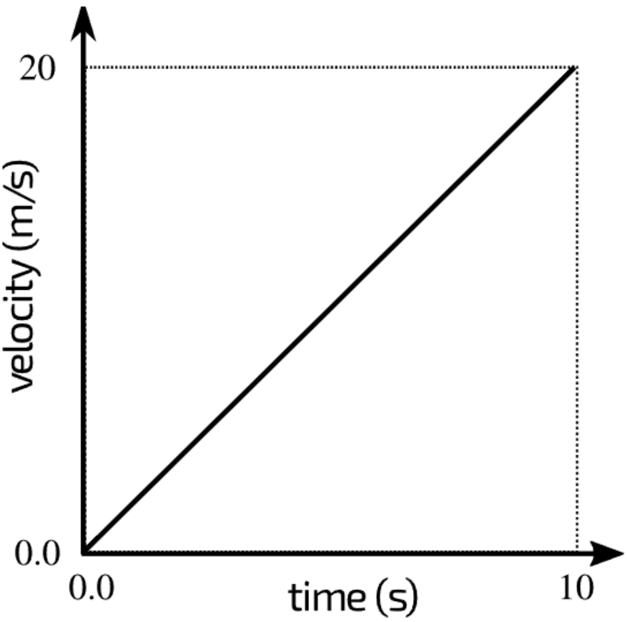
The acceleration between 10 to 15s is given by the gradient of the second line segment = $\frac{20-10}{15-10} = 2m/s^2$.

f. The acceleration between 15 to 20s.

Answer

The acceleration between 10 to 15s is given by the gradient of the second line segment = $\frac{0-20}{20-15} = -4m/s^2$.

(iv) Calculate the displacement moved and the acceleration for the following velocity-time graphs.



a. What is the displacement shown in the graph?

Answer

The displacement is give by the area under the curve which is $\frac{1}{2} \times 20 \times 10 = 100m$.

b. What is the acceleration?

Answer

The acceleration is given by the gradient is $\frac{20-0}{10-0} = 2m/s^2$.

4.6 Resultant Force and Acceleration

(i) A mass has weight of 1.0N.

a. If this is the only force on the mass, what is its acceleration?

Answer

$F = 1N = 0.1kg \cdot a \Longrightarrow a = 10m/s^2.$

b. What would be the weight of a 300g mass in the same gravitational field?

Answer

$F = 0.3kg \cdot 10m/s^2 \Longrightarrow F = 3N.$

c. If the weight is the only force on the 300g mass, what is its acceleration?

Answer

If weight is the only force, then $g = 10m/s^2$ is the only acceleration.

(ii) Complete the table. Each row describes a different object which has two forces acting upon it - one forwards (in the direction of motion), one backwards. Define forces and accelerations acting forwards as positive.

Answer

Force(N)			Mass(kg)	Acceleration(m/s^2)
Forwards	**Backwards**	**Resultant**		
58	16	**42**	5.6	**7.5**
90	145	**−55**	22	**−2.5**
1200	350	804	120	6.7

(i) Is the object in the first row speeding up or slowing down?

Answer

Speeding up as the acceleration is positive.

(ii) Is the object in the second row speeding up or slowing down?

Answer

Slowing down as the acceleration is negative.

(iii) Is the object in the third row speeding up or slowing down?

Answer

Speeding up as the acceleration is positive.

(iii) What unbalanced force acts on a 70kg mass if it accelerates at 1.6m/s^2?

Answer

From $F = ma$, we see that the magnitude of the unbalanced force is $112N$.

(iv) What is the acceleration of a 10kg mass which has no unbalanced force acting on it?

Answer

The acceleration is $0m/s^2$ as there is no unbalanced force.

(v) A 1200kg vehicle is accelerating along a straight road at 3.0m/s^2.What is the magnitude of the unbalanced force acting on it?

Answer

The magnitude of the unbalanced force is given by $F = ma$ which is $1200kg \times 3m/s^2 = 3600N$.

(vi) What force must I apply to a mass of 3.0kg to accelerate it at 4.0m/s^2 on a horizontal surface if:

a. There is no friction.

Answer

The force requires is given by $F = ma$ which is $3kg \times 4m/s^2 = 12N$.

b. There is a 4N friction.

Answer

If F is the required force, we have $F - 4N = 3kg \times 4m/s^2 = 12N \implies F = 16N$.

(vii) The thrust generated by a rocket engine is equal to the mass of propellant burnt each second multiplied by the exhaust velocity of the gas. The Space Shuttle (with booster rockets and external tank) had a total mass of 2040000kg at launch. In this question we shall assume that the exhaust velocity of the gas was 3000m/s, and we will take g=10N/kg.

a. How much propellant would have to be burnt each second in order for the spacecraft to hover just above the launch pad?

Answer

The weight of the rocket(force in the downward direction) would have to be balanced by the thrust (force in the upward direction) for the rocket to hover. The weight of the rocket is $mg = 2040000kg \times 10N/kg = 20400000N$. If m_p is the mass of the propellant burnt every second, then the upward thrust is $3000m/s \times m_p kg/s$. Therefore, $m_p = \frac{20400000}{3000} = 6800kg/s$.

b. How much propellant would have to be burnt each second in order for the spacecraft to accelerate upwards from the launch pad at "3g" (i.e. 30m/s^2)?

Answer

Let T be the total thrust required to accelerate the spacecraft upward to $3mg$. Therefore, we have $T - mg = 3mg \implies T = 4mg$, where m is the mass of the rocket. We also have $T = 3000 \cdot m_p N$. Therefore, $m_p = \frac{4 \cdot 2040000 \cdot 10}{3000} = 27200kg$.

(viii) A trained athlete runs a race. Her legs produce a constant force forwards. Her mass is 80kg. The instant she starts to run, her acceleration is 3.0m/s^2.

a. What is the maximum accelerating force provided by her legs?

Answer

The maximum accelerating force is $F = m \cdot a = 80kg \cdot 3m/s^2 = 240N$.

b. After a short time, her acceleration has fallen to 1.0m/s^2 yet she is not tired. At this instant, what is the air resistance?

Answer

Air resistance is a force acting in the opposite direction of motion. If the air resistance is f, the resultant force is $240N - f$N $= 80kg \cdot 1m/s^2$. Therefore the air resistance is $f = 240N - 80N = 160N$.

(ix) What is the frictional force if 3.0kg mass accelerates along a horizontal surface at 2.5m/s^2 when acted on by a pulling force of 10N?

Answer

Frictional force is a force acting in the opposite direction of motion. If the frictional force is f, the resultant force is $10N - f\text{N} = 3kg \cdot 2.5m/s^2$. Therefore the frictional force is $f = 10N - 7.5N = 2.5N$.

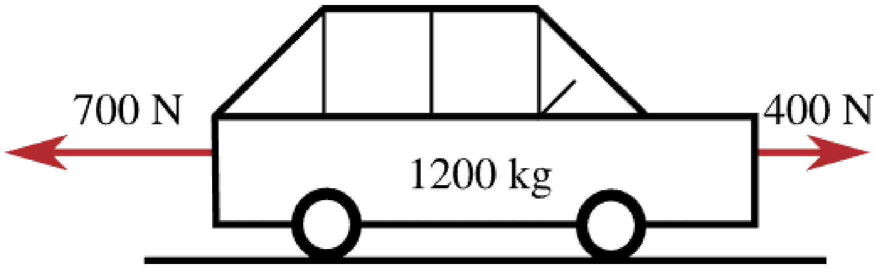
Answer

(x) A 1500kg car accelerates at 1.5m/s^2 along a horizontal road. If the frictional forces acting against the car's motion total 1000N, what driving force is exerted on the road by the car's wheels?

Answer

Let F N be the driving force acting on the car. We have
$F - 1000N = 1500kg \cdot 1.5m/s^2 \implies 3250N.$

(xi) The 1400kg car shown in Figure is travelling along a flat, straight road. Take the direction of the car's motion to be positive



a. Calculate the resultant force acting on the car.

Answer

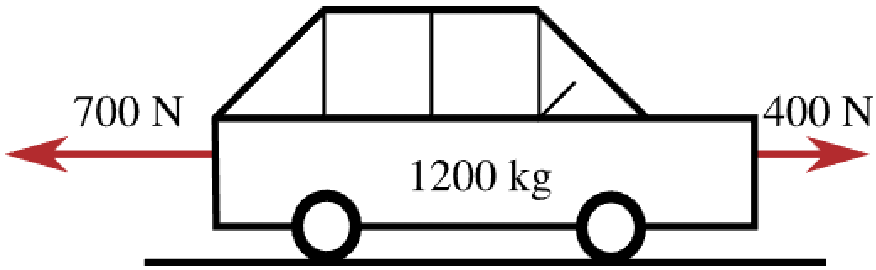
The resultant force acting on the car if $1200N - 500N = 700N$.

b. Calculate the car's acceleration with these forces acting on it.

Answer

The acceleration of the car is $\frac{700N}{1400kg} = 0.5m/s^2$.

(xii) The 1200kg car shown in Figure is travelling along a flat, straight road. Take the direction of the car's motion to be positive



a. Calculate the resultant force acting on the car.

Answer

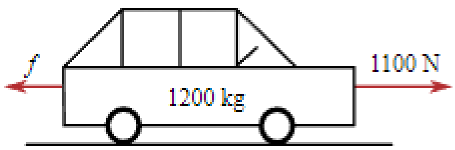
The resultant force acting on the car if $400N - 700N = -300N$.

b. Calculate the car's acceleration with these forces acting on it.

Answer

The acceleration of the car is $-\frac{300N}{1200kg} = -0.25m/s^2$.

(xiii) The car is travelling along a flat, straight road. Its mass is 1200kg and it is accelerating forward at 0.50m/s^2.



a. Calculate the resultant force acting on the car.

Answer

The resultant force acting on the car if
$1100N - f = 1200kg \cdot 0.5m/s^2 = 600N$.

b. Calculate the size of f, the friction force.

Answer

The size of f is $1100N - 600N = 500N$.

4.7 Terminal Velocity

(i) A parachutist of mass 80kg is falling to the ground at a steady vertical velocity of 10m/s.What is the value of the total drag force acting on him?

Answer

When the parachutist is falling to the ground at terminal velocity, the drag force is equal to weight. The drag force is $80kg \cdot 10m/s^2 = 800N$.

(ii) A bicycle is ridden along a horizontal road with a driving force of 400N. Its speed is constant at 12m/s. What is the magnitude of the sum of the frictional forces acting on the bike and its rider?

Answer

When the speed of the bike is constant, the resultant force on the bike is zero, i.e. the driving force is equal to the frictional forces. Therefore the magnitude of frictional forces is $400N$.

(iii) Two identical bottles are dropped off a cliff, both falling with their bases downwards. One of them is full of water, while the other contains air at atmospheric pressure.

Answer At the time a bottle reaches terminal velocity, the weight of the bottle is equal to drag force which is proportional to the square of speed.Therefore a bottle with more mass has more weight which means more drag force is acting on it which means higher speed. Therefore the bottle filled with water which has more mass will have a higher terminal velocity than that filled with air.

(iv) A car travelling at constant speed has a driving force of 2.1kN acting on it. The driver presses the accelerator, and the driving force increases to 2.5kN.

a. Immediately after the driver presses the pedal, what is the total resistive force acting on the car?

Answer

When the car is travelling at constant speed before the driver presses the accelerator, the resultant force is zero, therefore resistive force is equal to driving force which is $2.1kN$. The resistive force doesn't change immediately after the accelerator is pressed, therefore it is still $2.1kN$.

b. When the car again reaches a constant speed, what is the total resistive force acting on the car?

Answer

When the car again reaches a constant speed, the total resistive force acting on the car is equal to the new driving force which is $2.5kN$.

(v) A care package has a mass of 150kg. It is dropped from a helicopter 2000m above the ground.

a. What is the weight of the care package?

Answer

The weight of the package is $mg = 150kg \times 10m/s^2 = 1500N$.

b. If the object is falling at terminal velocity, what is the value of the air resistance?

Answer

If the object is falling at terminal velocity, the downward force of the weight is balanced by the upward force of air resistance. The value of the air resistance is therefore $1500N$.

c. The care package has a parachute built in, which opens automatically at 1000m. What happens to the air resistance as it opens?

Answer

The drag coefficient depends on the shape of the object as well as the density of the medium. The total air resistance(drag) increases at it opens because the parachute increases the surface area of the care package.

d. After a short time, the care package plus the open parachute begin to fall with its terminal velocity. How does this value compare to the terminal velocity of the care package without the parachute?

Answer

At terminal velocity, the drag is equal to the weight of the object. Let the drag coefficient of the package without the parachute be D_o and the drag coefficient with the parachute be D_p. $D_p > D_o$ because the parachute increases the surface area. Let the terminal velocity of the care package without the parachute be v_o and terminal velocity of the care package with the parachute be v_p. We have

$$mg = D_o v_o^2 = D_p v_p^2. \tag{4.1}$$

As $D_p > D_o$, we have $v_p < v_o$, i.e. the terminal velocity of the care package with parachute is smaller than that without.

e. How does the air resistance of the care package falling at terminal velocity with the parachute open compare to the air resistance of the care package falling at terminal velocity without the parachute? Explain your answer.

Answer

At terminal velocity, the total air resistance equals the weight of the object. Therefore the total air resistance doesn't change as the weight of the object is unchanged.

f. The care package falls off course and lands in a lake. Ignore the effect of buoyancy. The care package starts to sink and reaches terminal velocity once more. How does this value of terminal velocity qualitatively compare to the previous two values? Explain your answer.

Answer

As water is more dense and viscous than air, the drag for a given speed will be larger than that in air. Therefore the care package (together with its parachute) will have a lower terminal velocity in water than air.

g. what is the value of the fluid drag through the water when the object sinks at its terminal velocity?

Answer

When the object sinks at its terminal velocity in water, the drag is equal to it's weight which is $1500N$.

(vi) Figure below shows how the drag force on a 25000kg lorry depends on the lorry's speed.

(i) At one moment, the lorry is travelling at 20m/s and the driving force acting on it is 80000N.What is the acceleration of the lorry at this instant?

Answer

Let a be the instantaneous acceleration of the lorry. The drag force acting on the lorry when it is travelling at $20m/s$ if $20kN$. The resultant force acting on the lorry is
$80kN - 20kN = 60kN = 25000kg \cdot a$. Therefore $a = 2.4m/s^2$.

(ii) The lorry maintains a constant driving force of 80000N.

Answer

When the lorry reaches terminal velocity the drag on the lorry is equal to the driving force which is $80kN$. From the graph we see that the drag force is $80kN$ when the speed is $40m/s$.

(iii) While at the above terminal speed, the driver then halves the driving force to 40000N. What is the initial acceleration of the lorry?

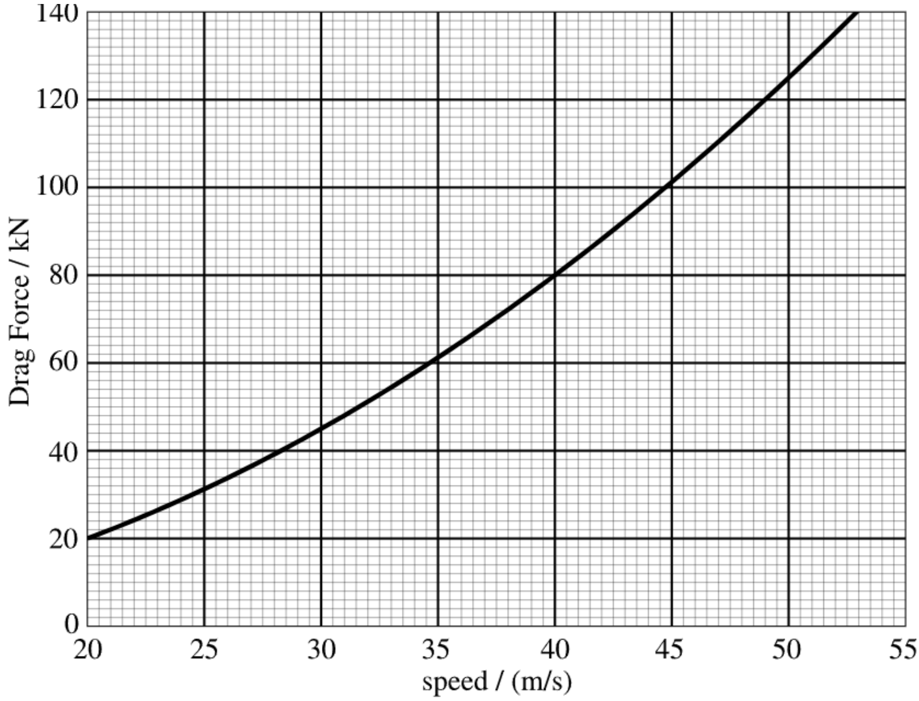
Answer

At the terminal velocity, the drag force is equal to the driving force which is $80kN$. When the driving force is halved the resultant force if $40kN - 80kN = -40kN$. The initial acceleration of the lorry if $-\frac{40kN}{25000kg} = -1.6m/s^2$.

(iv) After some time the lorry reaches a new terminal velocity. What is this new terminal velocity? Give your answer to 2 significant figures.

Answer

When the lorry reaches the new terminal velocity, the drag force acting on it is $40kN$. From the graph we see that the speed is $\approx 28m/s$ when the drag is $40kN$.



(vii) When an object is travelling through a fluid (liquid or gas), the resistive force will depend on the speed of the object. However, how this force changes with speed depends on whether the flow of the fluid past the object is smooth or turbulent.

a. If the flow is smooth, the resistive force is proportional to the speed of the object moving through the fluid. If the driving force is multiplied by 5, by what factor does the terminal velocity of the object increase?

Answer

If the driving force is multiplied by 5 and the object is moving through terminal velocity, that means that the resistive force has been multiplied by 5 which in turn means that the speed has to be five times the initial speed.

b. If the flow is turbulent, the resistive force is proportional to the square of the speed of the object moving through the fluid. If the driving force is multiplied by 5, by what factor does the terminal velocity of the object increase? Give your answer to 3 significant figures.

Answer

When the flow is turbulent, the driving force is multiplied by five and the object reaches terminal velocity, that means the resistive force is also multiplied by 5. As the resistive force is proportional to the square of the speed, the speed should be multiplied by a factor of $\sqrt{5} \approx 2.24$ if the resistive force becomes five fold.

(viii) When a ship is moving through the water, the resistance to its motion partly comes from making waves. This is called the "Wave Making Resistance". The other part of the resistance is known as "Frictional Resistance". For one ship, Frictional Resistance is 80% of the total resistance. The remainder comes from making waves. The frictional resistance is F_f, when the ship is at terminal velocity. Write an equation for F_D, the driving force propelling the ship, in terms of F_f.

Answer

Let F_w be the wave making resistance. When the ship is at terminal velocity, the driving force is equal to the total fricional force i.e. $F_D = F_f + F_w$. We also have $F_f = 0.8(F_f + F_w)$. Therefore we have $F_D = \frac{F_f}{0.8} = 1.25F_f$.

4.8 Stopping with and without brakes

The thinking distance at 30mph is 8.9m.

The braking distance at 30mph is 13.4m.

(i) Calculate the overall stopping distance from 30mph (according to the Highway Code).

Answer

The overall stopping distance is the sum of thinking and braking distance which is $8.9m + 13.4m = 22.3m$.

(ii) The Highway Code estimates that a typical reaction time of a driver is two thirds of a second; and that once applied, brakes will give a car a 6.67m/s^2 deceleration.

1mph=0.447m/s

a. Using the Highway code value for reaction time (given above), calculate the thinking distance at 60mph.

Answer

The thinking distance at $60mph$ is $60 \times 0.447 \times \frac{2}{3} \approx 18m$.

b. Using the Highway code value for deceleration (given above), calculate the braking distance at 60mph.

Answer

We have $v^2 - u^2 = 2as$. Here the final velocity is $0m/s$, the initial velocity is $60mph = 26.82m/s$ and acceleration is $-6.67m/s^2$.

Therefore breaking distance at $60mph$ is $\frac{-26.82^2}{2 \times -6.67} \approx 54m$.

c. Use Example 1 and your answer to (a) to complete the sentence: when you double your speed, the thinking distance...

Answer

The reaction time doesn't change with speed and thinking distance is directly proportional to speed. Therefore thinking distance doubles when speed doubles.

d. Use Example 2 and your answer to (b) to complete the sentence: when you double your speed, the braking distance...

Answer

Tne braking distance is directly proportional to square of the speed. Therefore braking distance quadruples when speed doubles.

(iii) A car is travelling at 70mph.

(i) Calculate the thinking distance.

Answer

The thinking distance is $\frac{2}{3}s \times 70 \times 0.447m/s \approx 21m.$

(ii) Calculate the braking distance.

Answer

The braking distance is $\frac{-(70\times0.447)^2}{2\times-6.67} \approx 73m.$

(iii) Another car is travelling at 35mph. What is its overall stopping distance?

Answer

(iv) A car runs into a wall and stops in 0.30s. It was going at a speed of 20m/s.

(i) Calculate the deceleration.

Answer

We have $v = u + at$. Here $v = 0$, $u = 20m/s$ and $t = 0.3s$. Therefore the deceleration is $67m/s^2$.

(ii) A person fixed to the car by a seatbelt has the same deceleration. They have a mass of 70kg. Calculate the force on the person.

Answer

We have $F = ma$. Here $m = 70kg$ and $a = 67m/s^2$. Therefore the force on the person is $70kg \times 67m/s^2 \approx 4700N$.

4.9 Moments, Turning and Balancing

(i) What force, acting at a distance of $25cm$ from the axis of rotation of a solid body, would make a moment of $10Nm$?

Answer

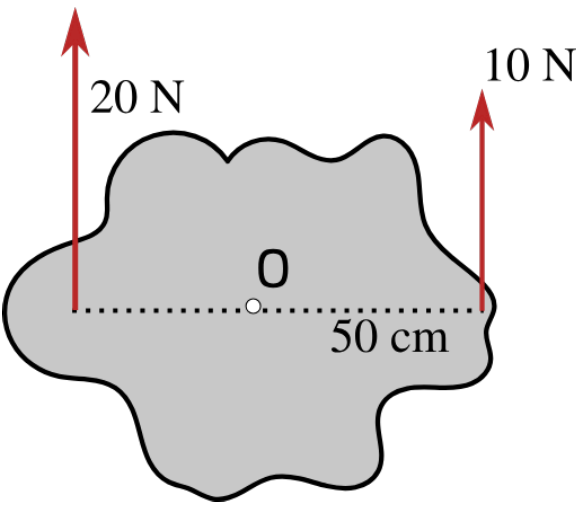
Let F be the required force in N. We have $0.25 \times F = 10 \Rightarrow F = 40N$.

(ii) What force, acting at a distance of $2.5m$ from the axis of rotation of a solid body, would make a moment of $25Nm$?

Answer

Let F be the required force in N. We have $2.5 \times F = 25 \Rightarrow F = 10N$.

(iii) Two forces act on a rigid body free to rotate about a perpendicular axis through point O. The sizes of the forces are $10N$ and $20N$.

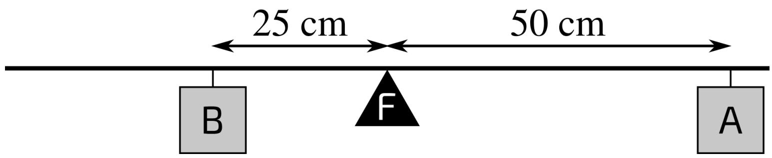


The perpendicular distance of the line of action of the $10N$ force from O is $50cm$. What is the perpendicular distance of the $20N$ force from O if the body does not rotate?

Answer

Let the required distance be $d$cm. If the body is not rotating, the net torque on the body is 0. Therefore, we have $50 \times 10 = 20 \times d \Rightarrow d = 25cm$.

(iv) Calculate the weight of the block stated in each situation below where the uniform lever arm is balanced about the fulcrum $'F'$. If A weighs $5N$, what is the weight of B?



Answer

As the lever is balanced, the net torque around F is 0. We have
$B \times 25 = 5 \times 50 \Rightarrow B = 10N$.

4.10 Pressure, Hydraulic Systems and Depth

(i) Assume the flat end of the drawing pin has an area of $1.2cm^2$.

  (i) Calculate the pressure on the person's finger when they are pushing in the pin with a force of $8.0N$.

  Answer

  The pressure on the person's finger is $\frac{8}{1.2} = 6.7$ N/cm^2.

  (ii) Give your answer in Pa.

  Answer

  $1Pa = 1\frac{N}{m^2} \Rightarrow 6.7\frac{N}{cm^2} = 6.7\frac{N}{0.0001m^2} = 67000Pa.$

(ii) My weight is $670N$, and each of my shoes has a sole area of $200cm^2$.

  (i) What will be the pressure when I stand on the ground?

  Answer

  The pressure when I stand on ground is $\frac{670}{2\times200cm^2} = 16750Pa.$

  (ii) A plank $15cm$ wide and $1.5m$ long is laid across a muddy path. What will be the pressure on the mud when I stand on the plank?

  Answer

  The pressure on the mud is $\frac{670}{0.15\times1.5m^2} = 2978Pa.$

  (iii) Compare your answers for (1) and (2). Can the plank stop me sinking into the ground?

  Answer

  The plank greatly reduces the pressure I apply to the ground by increasing the surface area over which my weight is distributed, which might be enough to prevent (or slow down) sinking.

  (iv) At a garage, a car ($8.0kN$ weight) is going to be lifted on four hydraulic jacks, each with a cross sectional area of $25cm^2$. Fluid is forced into the jacks by a compressor. If you want to support the car on the jacks, what is the pressure in the fluid?

  Answer

  The pressure on the fluid is $\frac{8000}{4\times25cm^2} = 800KPa.$

(iii) If needed, assume water has a density of $1000\frac{kg}{m^3}$, the gravitational field strength on Earth is 10N/kg and atmospheric pressure is $101kPa$. A watch states that it is 'water resistant to $30m$'.

(i) What extra pressure can it withstand before leaking?

Answer

The extra pressure at a depth of $30m$ under water is ρgh which is $1000 \times 10 \times 30 = 300KPa$.

(ii) What is the extra pressure on the watch at a depth of $10m$?

Answer

The extra pressure at a depth of $30m$ under water is ρgh which is $1000 \times 10 \times 10 = 100KPa$.

(iv) If needed, assume atmospheric pressure is $101kPa$.The deepest part of the Pacific Ocean, the Mariana Trench, has a depth of $10994m$.

(i) What is the total pressure at that depth? The density of sea water is $1030\frac{kg}{m^3}$.

Answer

The total pressure is atmospheric pressure + pressure due to sea water. Therefore the total pressure is
$101000 + 1030 \times 10 * 10994 = 113339200Pa$.

(ii) What would be the inwards force on a $10cm$ by $10cm$ window in a submarine at the pressure calculated in (i)?

Answer

The inward force on the window is $\frac{113339200}{100}\frac{N}{m^2} = 1133392kN$.

(v) Assume atmospheric pressure is $101kPa$. An ocean temperature probe is lowered from a survey ship into the water. The maximum pressure that the probe is designed to withstand is $100MPa$. What is the greatest depth to which the probe could be safely lowered? The density of sea water is $1030\frac{kg}{m^3}$.

Answer

Let dm be the greatest depth to which the probe can be safely lowered. The total pressure at that depth is
$101000 + 1030 \times 10 \times d = 100000000 \Rightarrow d = 9698m$.

4.11 Moving in a Circle

(i) A $0.50kg$ object travels in a $1.4m$ radius circle at a speed of $2.0\frac{m}{s}$.

  (i) Calculate the force needed to keep it in this motion.

  Answer

  The force needed to keep it in motion is given by
  $\frac{mv^2}{r} = \frac{0.5\times 2^2}{1.4} = 1.43N$

  (ii) Calculate the force needed for a circle of twice the radius with the same speed.

  Answer

  The force needed to keep it in motion is given by $\frac{0.5\times 2^2}{2\times 1.4} = 0.71N$

  (iii) Calculate the force needed for a speed twice as great for the original circle.

  Answer

  The force needed to keep it in motion is given by $\frac{0.5\times 4^2}{1.4} = 5.72N$

(ii) What force is needed to enable a $1525kg$ car travelling at $20mph(8.9\frac{m}{s})$ to go round a roundabout with an $8.0m$ radius?

Answer

The force needed is given by $\frac{1525\times 8.9^2}{8} = 15099N$

(iii) On a fairground ride, the passengers stand against a railing $5.3m$ from the centre of a large wheel.If the wheel rotates once every $2.3s$, what is the acceleration of the riders?

Answer

The acceleration is given by $r\omega^2$ where ω is the angular velocity. In this case $\omega = \frac{2\pi}{2.3}\frac{rad}{s}$. Therefore the acceleration is $5.3 \times \left(\frac{2\pi}{2.3}\right)^2 = 39.55\frac{m}{s^2}$.

(iv) A road over a humped-back bridge can be represented as a sector of a circle of radius $30m$. How fast could you travel over the top of the bridge before your wheels lifted off the ground there?

Answer

For the wheel to be lifted off the ground, the centrifugal force should be equal to the weight or the centrifugal acceleration should be equal to acceleration due to gravity. Let $v\frac{m}{s}$ be the required speed. We have $\frac{v^2}{30} = 10 \Rightarrow v = \sqrt{300} = 17.32\frac{m}{s}$.

(v) The formula for the gravitational field strength g (in N/kg) at distance r (in metres) from the Earth's centre is: $g = \frac{k}{r^2}$ where $k = 4.0 \times 10^{14} N\frac{m^2}{kg}$.

(i) Calculate the orbital speed of a satellite orbiting in a circle of radius $4.0 \times 10^7 m$ around the Earth.

Answer

For the satellite to be moving in a circle of constant radius around the earth, the acceleration due to gravity should be equal to the centrifugal acceleration. Let v be the orbital speed. We have $\frac{v^2}{r} = \frac{k}{r^2}$ where $r = 4.0 \times 10^7 m$. Therefore $v = \sqrt{\frac{k}{r}} = 1000\sqrt{10} = 3162\frac{m}{s}$.

(ii) Derive a formula for the time for one orbit of a satellite around the Earth in terms of the radius of its orbit. Use k rather than a numeric value in your formula.

Answer

The orbit time is given by $\frac{2\pi r}{v} = \frac{2\pi r}{\sqrt{\frac{k}{r}}} = \frac{2\pi}{\sqrt{k}} r^{\frac{3}{2}}$.

(iii) At what radius of orbit will a satellite orbit once every 24hours?

Answer

Using the orbit time formula from the previous question we can calculate the radius for which the orbit time is 24 hours. We have $\frac{2\pi}{\sqrt{k}} r^{\frac{3}{2}} = 24 \cdot 3600 \Rightarrow r = 4.2 \cdot 10^7 m$

(vi) The satellites which provide us with satellite TV are geostationary. Why are the TV companies willing to spend the extra money to put a satellite so far away from the Earth, when a nearer one would be much cheaper?

Answer

Because otherwise we would lose TV signal as they orbit around the Earth.

4.12 Introducing Momentum and Impulse

(i) A $5.0kg$ trolley is initially moving at $3.5\frac{m}{s}$ West. A $12.4N$ force (East) acts on it for $3.5s$. Take 'travelling East' as being positive, and 'travelling West' as being negative.

(i) Calculate the acceleration.

Answer

The acceleration is $\frac{12.4}{5} = 2.48\frac{m}{s^2}$.

(ii) Calculate the velocity change.

Answer

The initial velocity is $-3.5\frac{m}{s}$. The final velocity is $-3.5 + 2.48 \times 3.5 = 5.18\frac{m}{s}$. The change in velocity is $5.18 - (-3.5) = 8.68\frac{m}{s}$.

(iii) Calculate the original momentum.

Answer

The original momentum is $5 \times (-3.5) = -17.5kg(\frac{m}{s})$.

(iv) Calculate the new momentum.

Answer

The new momentum is $5 \times (5.18) = 25.9kg(\frac{m}{s})$.

(v) Calculate the change in momentum.

Answer

The change in momentum is $25.9 - (-17.5) = 43.4kg(\frac{m}{s})$.

(vi) Is the change in momentum equal to the product of the force and time?

Answer

Yes.the product of force and time is $12.4 \times 3.5 = 43.4kg(\frac{m}{s})$ which is the same as change in momentum.

(ii) What magnitude of force is needed to accelerate a $300000kg$ wide-body jet from $0.0\frac{m}{s}$ to take off speed of $90\frac{m}{s}$ in $50s$?

Answer

The acceleration of the jet is $\frac{90-0}{50} = 1.8\frac{m}{s^2}$. The force required is $300000 \times 1.8 = 540000N$

(iii) What will the momentum of a 200kg rocket be after a $10kN$ force has pushed it for four minutes?

Answer

The acceleration of the rocket is given by $\frac{10000}{200} = 50\frac{m}{s^2}$. The change in velocity during 4 minutes is $50 \times 240 = 12000\frac{m}{s}$. Therefore the momentum of the rocket will be $200 \times 12000 = 2400000kg(\frac{m}{s})$.

(iv) At what speed is a 20gram air rifle pellet moving if it has a momentum of 1.6kgm/s?

Answer

The speed of the pellet is $\frac{1600}{20} = 80\frac{m}{s}$.

(v) A girl on a $10kg$ bicycle is riding it at a speed of $6.0\frac{m}{s}$. If the momentum of the girl and bicycle is $360kg(\frac{m}{s})$, what is the mass of the girl?

Answer

Let the mass of the girl be $m$kg. We have $(m+10)6 = 360 \Rightarrow m = 50kg$.

(vi) An $800kg$ car is travelling at $70mph$ and overtaking a $15000kg$ truck travelling at $55mph$. Calculate the ratio of the momentum of the truck to the momentum of the car ($\frac{p_{truck}}{p_{car}}$).

Answer

The ratio of the momentum of the truck to the momentum of the car is $\frac{15000\times55}{800\times70} = 14.7$.

(vii) Calculate the momentum of a 20000tonne ship moving through the water at a speed of $12\frac{m}{s}$. [Note: $1tonne = 1000kg$.]

Answer

The momentum of the ship is give by $20000 \times 1000 \times 12 = 24 \times 10^7 kg(\frac{m}{s})$

(viii) A car is travelling at $15\frac{m}{s}$. It has $18000kg(\frac{m}{s})$of momentum. What is the car's mass?

Answer

The mass of the car is given by $\frac{18000}{15} = 1200kg$.

(ix) Two cars are travelling in the same direction. One has a mass of $1000kg$ and is moving at $10\frac{m}{s}$, the other's mass is $1200kg$ and it is moving at $15\frac{m}{s}$. What is the total momentum of the cars?

Answer

The total momentum of the cars is given by
$1000 \times 10 + 1200 \times 15 = 28000kg(\frac{m}{s})$.

4.13 Momentum Conservation

(i) Positive momentum means 'travelling East'. Negative momentum means 'travelling West'.

Calculate the momentum of:

a. A 3kg trolley moving at $2m/s$ to the East.

Answer

The momentum is given by $mv = 3kg \times 2m/s = 6kgm/s$.

b. A 700kg car moving at $6m/s$ to the West.

Answer

The momentum is given by $mv = 700kg \times -6m/s = -4200kgm/s$.

c. A 50g mass moving at 50cm/s to the East.

Answer

The momentum is given by $mv = 0.050kg \times 0.5m/s = 0.025kgm/s$.

d. A 1000g mass moving at 50cm/s to the East.

Answer

The momentum is given by $mv = 0.050kg \times 0.5m/s = 0.025kgm/s$.

e. A 10000kg bus moving Eastwards at 3.0m/s.

Answer

The momentum is given by $mv = 10000kg \times 3m/s = 30000kgm/s$.

f. What is the total momentum of the car in (b) and the bus in (d)?

Answer

The total momentum is $-4200 + 30000 = 25800kgm/s$.

g. If the car and the bus were to collide and stick together, what would the total mass be just after the impact?

Answer

The total mass after impact is $700 + 10000 = 10700kg$.

h. What would the total momentum of the car and bus just after the collision?

Answer

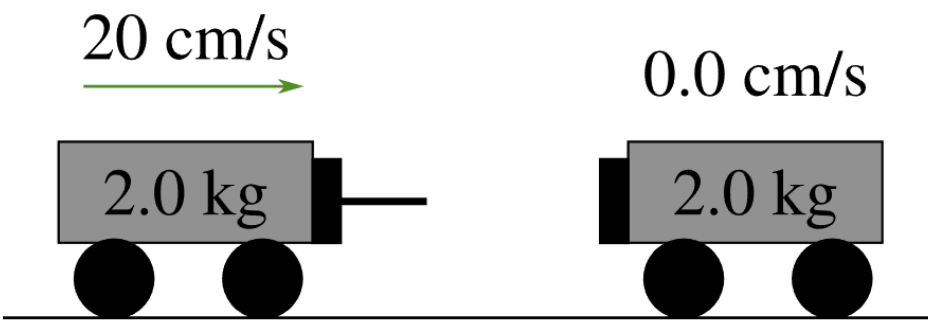
The initial momentum of the bus and car before collision is $-4200 + 30000 = 25800 kgm/s$. Therefore the total momentum of the car and bus after collision is also $25800 kgm/s$ as momentum is conserved.

i. Calculate the initial velocity of the car and bus just after they have stuck together.

Answer

The total momentum after collision is $25800 kgm/s$. The total mass after collision is $10700 kg$. Therefore the velocity after collision is $\frac{25800}{10700} \approx 2.4 m/s$.

(ii) Two 2.0kg trolleys collide and stick together on a smooth, horizontal surface. One trolley is at rest before the collision and the other is initially moving at 20cm/s, as shown in the figure.

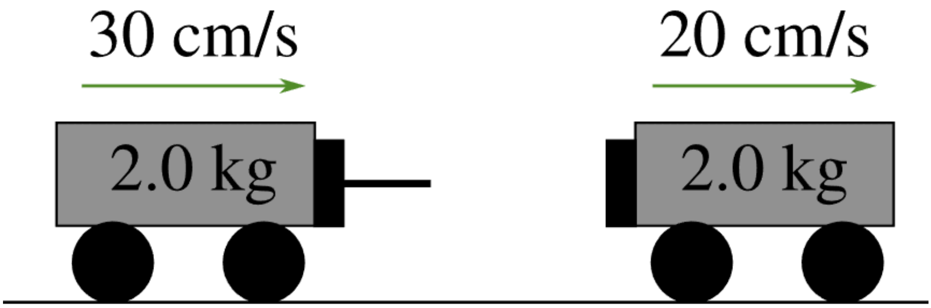


Calculate the combined velocity of the trolleys after the collision.

Answer

Positive momentum means 'travelling East'. The initial momentum of the trolleys is $2kg \times 0.2m/s + 2kg \times 0.2m/s = 0.4 kgm/s$. The total mass of the trolleys after collision is $4kg$. As momentum is conserved, the momentum after collision is also $0.4 kgm/s$. Therefore, the velocity after collision is $\frac{0.4}{4} = 0.1 m/s$.

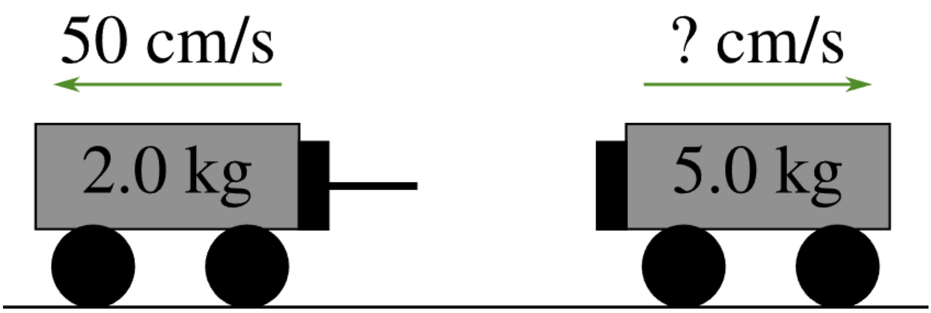
(iii) Two trolleys are moving in the same direction along a smooth surface. One is moving faster and catches up on the other. The trolleys collide and stick together.



Answer

Positive momentum means 'travelling East'. The initial momentum of the trolleys is $2kg \times 0.3m/s + 2kg \times 0.2m/s = 1kgm/s$. The total mass of the trolleys after collision is $4kg$. As momentum is conserved, the momentum after collision is also $0.4kgm/s$. Therefore, the velocity after collision is $\frac{1}{4} = 0.25m/s$.

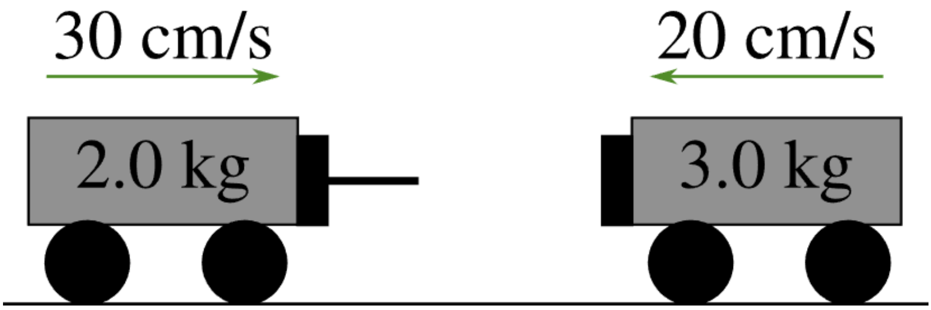
(iv) Two trolleys are at rest and in contact on a smooth, level surface. A coiled spring in one trolley is released so that they 'explode' apart. The lighter trolley moves off at 50cm/s.



Answer

Positive momentum means 'travelling East'. The initial momentum of the trolleys is $0kgm/s$ as they are at rest. Let vm/s be the velocity of the other trolley. The momentum after the spring is released is given by $2 \times -0.5 + 5 \times v = (-1 + 5v)kgm/s$. As momentum is conserved, we have $0 = -1 + 5v \implies v = 0.2m/s$.

(v) Two trolleys are moving in opposite directions along a smooth surface. The trolleys collide and stick together.



a. What is the total momentum of the trolleys before the collision?

Answer

Positive momentum means 'travelling East'.

The initial momentum of the trolleys is
$\mathbf{2 \times 0.3 + 3 \times (-0.2) = 0} kgm/s$.

b. What is the total momentum of the trolleys after the collision?

Answer

As momentum is conserved, the total momentum after collision is also $0kgm/s$.

c. What is the trolleys' combined velocity after the collision?

Answer

As momentum after collision is $0kgm/s$, the combined velocity of the trolleys is $0m/s$.

(vi) Complete the following table for two objects A and B, travelling together until pushed apart by explosions.

m_A(kg)	m_B (kg)	**Initial velocity(m / s)**	**Final** v_A(m /s)	**Final** v_B(m/s)
2.5	2.5	0.0		2.0
2.5	5.0	0.0		
2.5	7.5	5.0		
9.0		−4.0		6.0

(vii) In this question:

Positive momentum means 'travelling East'. Negative momentum means 'travelling West'.

A 10g bullet is fired at 250m/s Eastwards towards a 10kg sandbag.

a. Calculate the momentum of the bullet.

Answer

The momentum of the bullet is $0.01 \times 250m/s = 2.5kgm/s$.

b. What is the total momentum before the collision?

Answer

The total momentum before collision is equal to the momentum of the bullet which is $0.01 \times 250m/s = 2.5kgm/s$ as the sandbag is at rest.

c. What is the total momentum now once the bullet enters the sandbag and stops inside it?

Answer

As momentum is conserved, the total momentum after collision is also $2.5kgm/s$.

d. Calculate the speed of the sandbag after the collision.

Answer

The total momentum after collision is $2.5kgm/s$. The mass of the sandbag after collision is $10.01kg$.Therefore speed of the sandbag after collision is $2.\frac{5}{10.01} \approx 0.25m/s$

(viii) In this question:

Positive momentum means 'travelling East'.

Negative momentum means 'travelling West'.

A 70kg astronaut has a 20kg backpack, and is stranded, stationary, in space 30m to the West of her spacecraft. To get back to safety, she hurls the backpack at a speed of 4.2m/s.

a. Which way does she need to throw the backpack?

Answer

The astronaut needs to travel East towards her craft, so she must throw her backpack to the West. By Newton's 3rd Law, the backpack will apply an Eastward force on her.

b. What is the total momentum before she throws it?

Answer

The astronaut is initially at rest, therefore the total momentum is $0kgm/s$.

c. What is the momentum of the backpack after throwing?

Answer

The momentum of the backpack after throwing is $20kg \times -4.2m/s = -84kgm/s$.

d. What will the astronaut's momentum be after she has thrown the backpack?

Answer

As momentum is conserved, the astronaut's momentum will be $84kgm/s$.

e. What is the astronaut's velocity after she has thrown the backpack?

Answer

The astronaut's velocity will be $\frac{84}{70} = 1.2m/s$.

f. How much time does it take her to get back to the spacecraft?

Answer

The takes $\frac{30}{1.2} = 25s$ to get back to her spacecraft.

(ix) The exhaust from a rocket on a test rig leaves the engine at 2800m/s.How many kilograms of propellant (fuel and oxidizer) need to be burnt every second to provide a force of $3.5 \times 10^8 N$?

Answer

Force is given by rate of change of momentum. Let m_p be the mass of propellant burnt every second. Therefore, $\frac{m_p \times 2800}{1} = 3.5 \times 10^8 \implies m_p = 1.25 \times 10^5 kg$.

(x) A conveyor belt is used to move coal along a horizontal shaft in a coal mine.How much force needs to be applied horizontally to the belt to keep it moving at 1.2m/s if 40kg of coal is dropped onto it every second?

Assume that the coal has no horizontal velocity before it touches the belt and the belt's turning mechanism is well lubricated.

Answer

Force is given by rate of change of momentum. The initial velocity of coal is $0m/s$. The final velocity of coal is $1.2m/s$. The rate of change of momentum is $\frac{40(1.2-0)}{1} = 48N$.

4.14 Motion with Constant Acceleration

(xi) A £5 note is 135mm long. A friend has a £5 note, and holds the bottom of the note in line with (and between) your thumb and index finger. She drops it, and if you grab it without moving your hand downwards, you are allowed to keep it. (Acceleration due to gravity g=10m/s^2).

Answer

You need to react before the note travels $135mm$. The initial speed of the note is $0m/s$. Let t be the time taken by the note to travel $135mm$.

Therefore we have $0.135 = \frac{1}{2}gt^2 \implies t = \sqrt{\frac{2\times 0.135}{10}} \approx 0.16s$.

(xii) The Highway Code assumes that a car with its brakes on fully has an acceleration of −6.7m/s^2.

(i) Calculate the time taken to stop a car from 30mph (13.4m/s).

Answer

We have $u = 13.4m/s$ and $a = -6.7m/s^2$.Therefore,
$t = \frac{0-13.4}{-6.7} = 2s$.

(ii) Calculate the distance taken to stop a car at 30mph (13.4m/s).

Answer

The distance is given by $\frac{0-13.4^2}{2\times(-6.7)} = 13.4m$

(iii) Calculate the time taken to stop a car from 70mph (31m/s).

Answer

We have $u = 31m/s$ and $a = -6.7m/s^2$.Therefore,
$t = \frac{0-31}{-6.7} \approx 4.6s$.

(iv) Calculate the distance taken to stop a car from 70mph (31m/s).

The distance is given by $\frac{0-31^2}{2\times(-6.7)} \approx 72m$

(xiii) You throw a cricket ball up into the air at 10m/s.

a. How much time elapses before it reaches the highest point of its motion?

Answer

At the highest point $v = 0m/s$. Let upward direction be positive. We have $u = 10m/s, v = 0m/s$ and $g = -10m/s^2$. Therefore $t = 1s$.

b. How high does it go?

Answer

The distance travelled by the ball in $1s$ is given by
$s = 10 \times 1 - \frac{1}{2} \times 10 \times 1 = 5m$.

(xiv) If there were no air resistance, how much time would it take for a dropped parcel to fall 2000m?

Answer

We have $2000 = \frac{1}{2} \times 10 \times t^2 \implies t = 20s$.

(xv) What is the deceleration of a train which takes 2.3km to stop from a speed of 67m/s?

Answer

We have $u = 67m/s$ and $v = 0m/s$. Therefore,
$0 - 67^2 = 2 \times a \times 2300 \implies a = -0.98m/s^2$.

(xvi) How much time does it take to stop an oil tanker if its speed is 8.0m/s to start with, and the stopping distance is 5.0miles? One mile is about 1600m.

Answer

Let t be the time taken by the tanker to stop. The initial speed of the tanker is $u = 8m/s$. The final speed is $0m/s$. The deceleration is given by $-\frac{8}{t}$. Therefore we have
$8000 = 8 \times t + \frac{1}{2} \times \left(-\frac{8}{t}\right) \times t^2 \implies t = 2000s$.

(xvii) The Eiffel Tower is 300m high. If a coin is dropped from the top, how fast is it going when it hits the pavement? Assume no air resistance.

Answer

Let the downward direction be positive. Let v be the final velocity of the coin. The initial velocity of the coin is $0m/s$. Therefore,
$v^2 - u^2 = 2 \times 10 \times 300 \implies v \approx 78m/s$.

(xviii) How fast would you have to shoot a scientific instrument upwards if you wanted it to rise 200km above the Earth's surface ignoring air resistance?

Answer

Let the upward direction be positive. Let u be the initial velocity. The speed of the instrument at the highest point $s = 200km$ is $0m/s$. Therefore, $0 - u^2 = -2 \times 10 \times 200000 \implies u = 200m/s$. One thing to note is that g is not constant over such a large distance. In fact, at 200km, g=9.2m/s^2.

(xix) The acceleration of dropped objects on the Moon is $1.6m/s^2$. How long does it take a feather to fall 0, point, 70, m,0.70m? [There is no air resistance!]

Answer

The initial speed of the feather is $0m/s$. Let t be the time taken by the feather to travel $0.7m$.

Therefore we have $0.7 = \frac{1}{2}1.6t^2 \implies t = \sqrt{\frac{2\times0.7}{1.6}} \approx 0.94s$.

5 Electricity

5.1 Charge and Current

Assume the electron has a charge of $-1.60 \times 10^{-19}C$.

(i) A 3.00A appliance has 360C of charge flow through it.

a. How long was the appliance operating?

Answer

We have $C = I \times t$ where C is the charge, I is the current and t is the time. Therefore, the appliance was operating for $\frac{360}{3} = 120s$.

b. How many electrons passed through the appliance in this time?

Answer

The number of electrons that passed through the appliance is given by $\frac{360}{1.60 \times 10^{-19}} = 225 \times 10^{19}$.

(ii) −1.00 C is the charge of how many electrons? Give your answer to 3 SF.

Answer

The number of electrons is $\frac{-1}{-1.60 \times 10^{-19}} = 0.625 \times 10^{19}$.

(iii) If two electrons are removed from an atom, what is the charge of the resulting cation (positively charged ion)?

Answer

The magnitude of charge due to two electrons is 3.2×10^{-19}.

(iv) An appliance draws a current of 9.00A.

a. How much charge flows in 5.0minutes of operation?

Answer

The amount of charge in 5 minutes is $5 \times 60 \times 9 = 2700C$.

b. How many electrons flow in 5.0minutes?

Answer

The number of electrons is $\frac{2700}{1.60 \times 10^{-19}} \approx 1.7 \times 10^{22}$.

(v) A car battery is rated at 40Ah.For how long could the battery power a headlamp bulb which draws 2.0A?

Answer

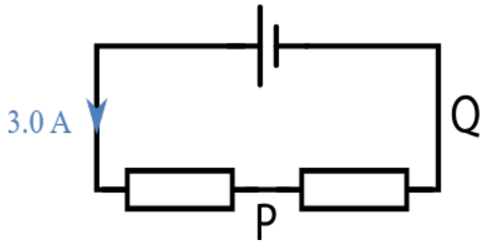
The battery can power the headlamp for $\frac{40Ah}{2A} = 20hrs.$

(vi) **Answer**

5.2 Additional Charge and Current Questions

5.3 Current and Voltage - Circuit Rules

(i)



(i) What is the current at P?

Answer

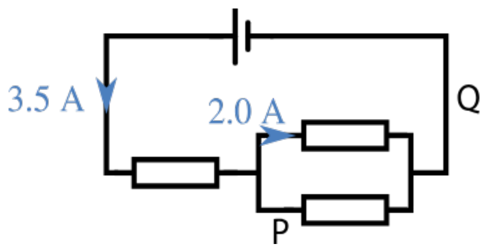
The current at P is $3A$.

(ii) What is the current at Q?

Answer

The current at Q is $3A$.

(ii)



(i) What is the current at P?

Answer

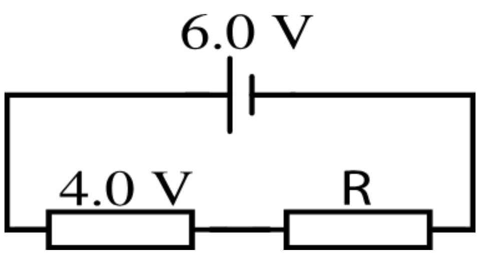
The current at P is $3.5 - 2 = 1.5A$.

(ii) What is the current at Q?

Answer

The current at Q is $3.5A$.

(iii)

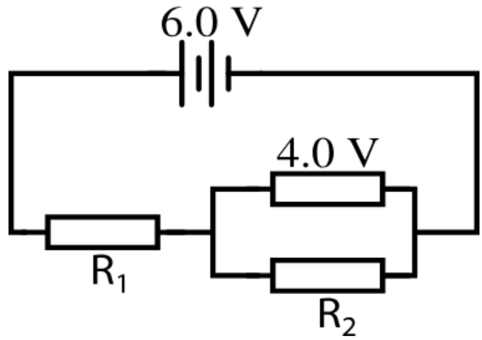


(i) What is the potential difference across resistor R?

Answer

The potential difference across R is $6 - 4 = 2V$.

(iv)



(i) What is the potential difference across resistor R_1?

Answer

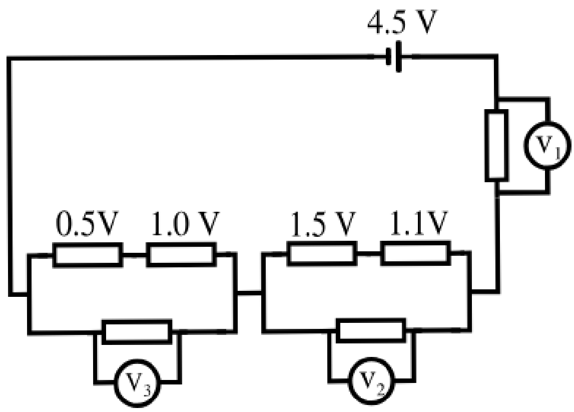
The potential difference across R_1 is $6 - 4 = 2V$.

(ii) What is the potential difference across resistor R_2?

Answer

The potential difference across R_2 is $4V$.

(v) In the circuit below, what are the readings on the voltmeters V_1, V_2 and V_3?



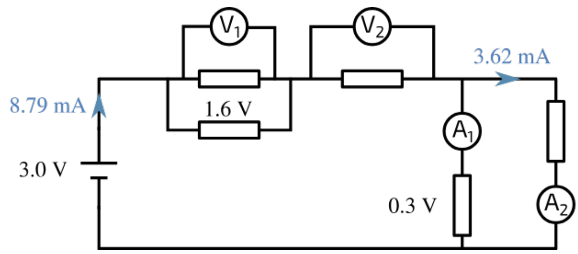
Answer

The reading on voltmeter V_1 is $4.5 - (0.5 + 1) - (1.5 + 1.1) = 0.4V$.

The reading on voltmeter V_2 is $1.5 + 1.1 = 2.6V$.

The reading on voltmeter V_3 is $0.5 + 1 = 1.5V$.

(vi)



(i) What is the reading on the voltmeter V_1?

Answer

The reading on voltmeter V_1 is $1.6V$.

(ii) What is the reading on the voltmeter V_2?

Answer

The reading on voltmeter V_1 is $3 - 1.6 - 0.3 = 1.1V$.

(iii) What is the reading on the voltmeter A_1?

Answer

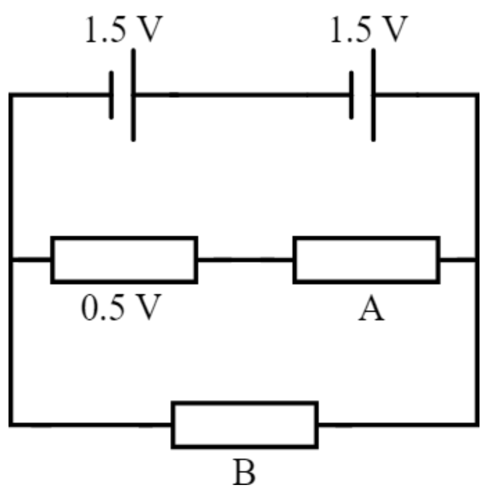
The reading on ammeter A_1 is $8.79 - 3.62 = 5.17mA$.

(iv) What is the reading on the voltmeter A_2?

Answer

The reading on ammeter A_2 is $3.62mA$.

(vii) In the circuit below, what are the missing voltages A and B?

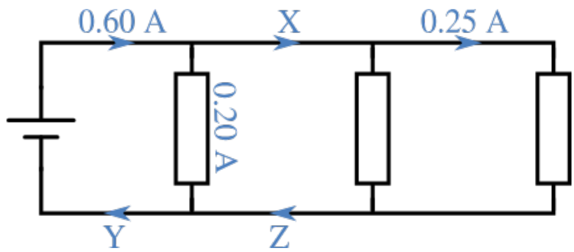


Answer

The missing voltage at B is $1.5 + 1.5 = 3V$.

The missing voltage at A is $1.5 + 1.5 - 0.5 = 2.5V$.

(viii) In the circuit below, what are the missing currents X, Y and Z?



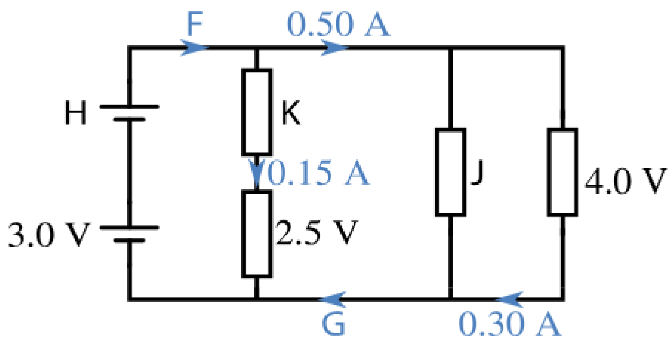
Answer

The missing current at X is $0.6 - 0.2 = 0.4A$.

The missing current at Y is $0.6A$.

The missing current at Z is $0.4A$.

(ix) In the circuit below, what are the missing currents (F and G) and voltages (H, J, K)?



Answer

The missing current at F is $0.5 + 0.15 = 0.65A$.

The missing current at G is $0.5A$.

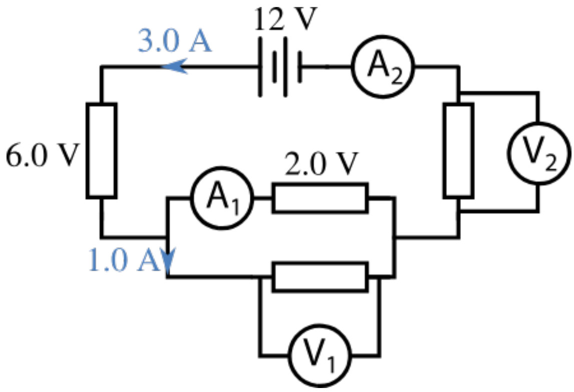
The missing current at Z is $0.4A$.

The voltage H is $4 - 3 = 1V$.

The voltage J is $4V$.

The voltage K is $4 - 2.5 = 1.5V$.

(x) In the circuit below, what are the readings on the ammeters (A_1 and A_2) and the voltmeters (V_1 and V_2)?



Answer

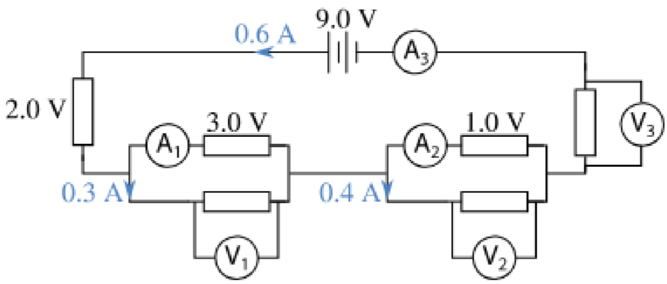
The reading on ammeter A_1 is $3 - 1 = 2A$.

The reading on ammeter A_2 is $3 = 3A$.

The voltage V_1 is $2V$.

The voltage V_2 is $12 - 6 - 2 = 4V$.

(xi) In the circuit below, what are the readings on the ammeters (A_1, A_2 and A_3) and the voltmeters (V_1, V_2 and V_3)?



Answer

The reading on ammeter A_1 is $0.6 - 0.3 = 0.3A$.

The reading on ammeter A_2 is $0.6 - 0.4 = 0.2A$.

The reading on ammeter A_3 is $0.6A$.

The voltage V_1 is $3V$.

The voltage V_2 is $1V$.

The voltage V_3 is $9 - 2 - 3 - 1 = 3V$.

5.4 Resistance

(i) What is the resistance of the heating element in an electric oven which carries a current of $10A$ when connected to $230V$ mains?

Answer

We have $V = IR$. Therefore, the resistance of the heating element is $\frac{230}{10} = 23\Omega$.

(ii) What resistance is needed if you wish to have a $10mA$ current and the supply voltage is $20V$?

Answer

The resistance needed is $\frac{20}{.}01 = 2000\Omega$.

(iii) A car headlamp bulb has a filament resistance of 6.0Ω. If the car battery is 13.2volts, how much current does the bulb take from the battery when lit?

Answer

The current taken by the bulb is $\frac{13.2}{6} = 2.2A$.

(iv) The heater of a toilet hand drier uses 9.0amps from a 240volt mains supply. What is the resistance of the heater's element?

Answer

The resistance of the heater's element is $\frac{240}{9} = 26.66\Omega$.

(v) Some resistors are labelled with a red band. This shows that their true resistance will be within 2.0% of the value shown on the side. What is the largest current you would expect through a $15k\Omega$, 2.0% resistor when connected to a $5.0V$ supply? Give your answer in mA.

Answer

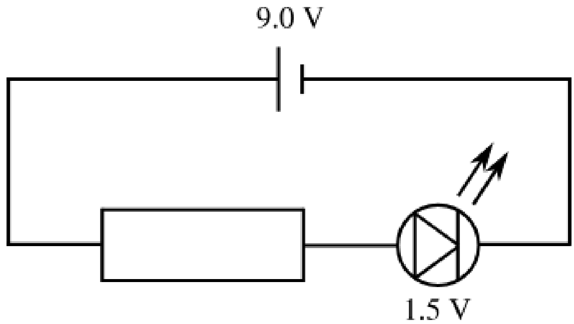
The resitance can vary between $15 \times 1.02k\Omega$ to $15 \times 0.98k\Omega$. The largest current that can be expected is when the resistance is lowest. Therefore the largest current through the resistor is $\frac{5}{15 \times 0.98 \times 1000} = 0.34mA$

(vi) When both of a car's red tail lamps are lit, this 'draws' a current of $0.83A$ from the battery. Given that a car battery has a voltage of $12V$, what is the resistance of each lamp? The lamps are wired in parallel.

Answer

The current going through each lamp is $\frac{0.83}{2}A$. as they are wired in parallel. The resistance of each lamp is $\frac{24}{0.83} = 28.91\Omega$.

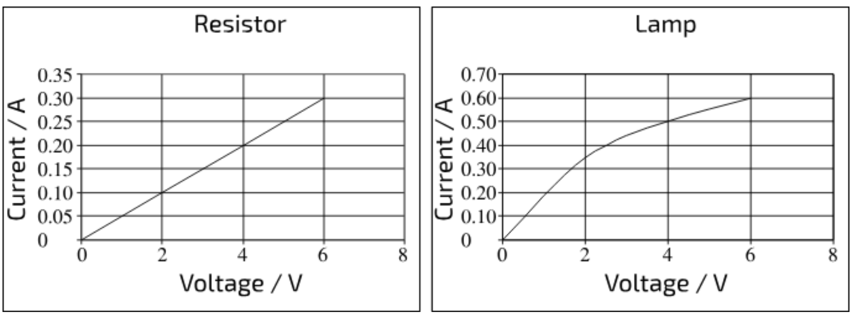
(vii) When a light emitting diode (LED) is connected into an electric circuit, it is wired in series with a resistor to ensure it doesn't get damaged by taking too much current. The voltage across a red LED is $1.5V$ when it is lit. You wish to supply it using a $9.0V$ battery.



Answer

The voltage across the resistor is $9 - 1.5 = 7.5V$. The required resistance is $\frac{7.5}{30\times10^{-3}} = 250\Omega$.

(viii) Study the two graphs below, showing the current passing through a resistor and lamp for different supply voltages.



(i) What is the resistance of the resistor when it is connected to a $6V$ supply?

Answer

The resitance is $\frac{6}{0.3} = 20\Omega$.

(ii) What is the resistance of the resistor when it is carrying $0.20A$?

Answer

The resitance is $\frac{4}{0.2} = 20\Omega$.

(iii) What current would you expect to flow through the resistor if the voltage across it were $\mathbf{10V}$?

Answer

As current is proportional to voltage based on the linear graph, the expected current is $\frac{10}{20} = \mathbf{0.5A}$.

(iv) What is the resistance of the lamp when it is connected to a $\mathbf{4V}$ supply?

Answer

The resistance of the lamp is $\frac{4}{0.5} = \mathbf{8\Omega}$.

(v) If you wired the resistor and the lamp in parallel and connected them to a $\mathbf{6V}$ supply, how much current would be 'taken' in total from the supply?

Answer

The resistance of the resistor is $\mathbf{20\Omega}$ and the resistance of the lamp is $\mathbf{10\Omega}$. The combined resistance when they are in parallel is $\frac{20\times10}{20+10} = \frac{20}{3}\mathbf{\Omega}$. The current that would be taken is $\frac{6}{\frac{20}{3}} = \mathbf{0.9A}$.

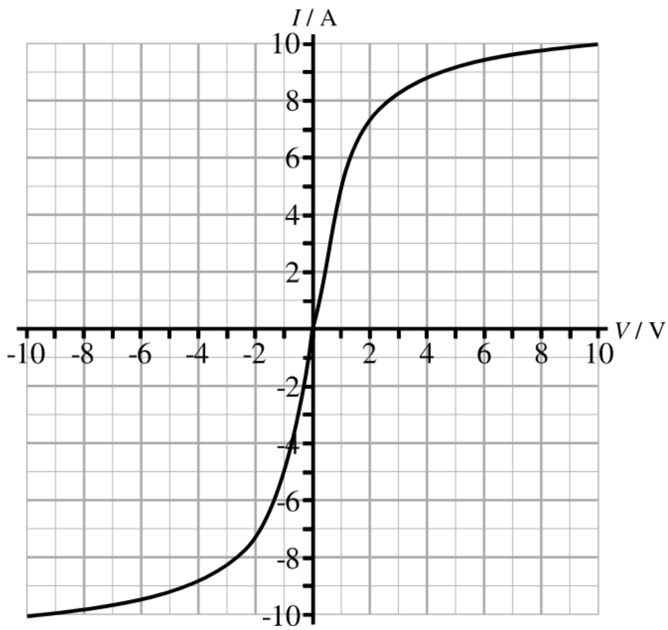
(vi) If you wired the resistor and the lamp in series, and connected this to a $\mathbf{14V}$ supply, what would the current be and the voltage across the lamp?

Answer

The resitor has a constant resistance of $\mathbf{20\Omega}$. The resistance of the lamp varies with voltage. Let the resitance of the lamp be $\boldsymbol{R\Omega}$ and the current flowing through the circuit be c. By trial and error, we can see that when the current is $\mathbf{0.5A}$, the voltage across the resistor is $\mathbf{20 \times 0.5 = 10V}$ which means that the voltage across the lamp is $\mathbf{4V}$. This current voltage relationship matches the voltage-current graph of the lamp.

5.5 Characteristics

(i) For these questions consider the following graph. When reading values from the axes, round to the nearest integer.



(i) What current is drawn at a voltage of $2V$?

Answer

From the graph, we can see that the current drawn at $2V$ is $7A$.

(ii) What current is drawn at a voltage of $6V$?

Answer

From the graph, we can see that the current drawn at $6V$ is $9A$.

(iii) What is the resistance at $1V$?

Answer

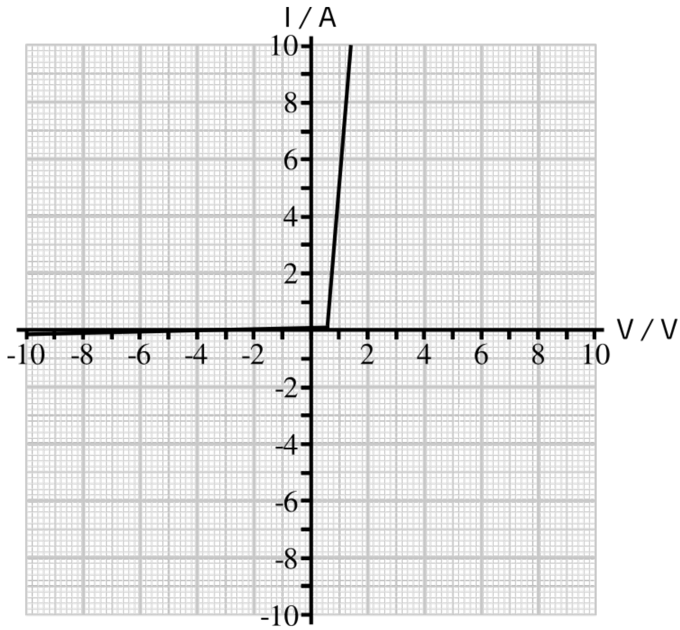
The current at $1V$ is $5A$. Therefore, the resistance is $\frac{1}{5} = 0.25\Omega$.

(iv) What is the resistance at $10A$?

Answer

The voltage at $10A$ is $10V$. Therefore, the resistance is $\frac{10}{10} = 1\Omega$.

(ii) The following characteristic graph is for a typical diode.



(i) What is the resistance at $1V$?

Answer

The current at $1V$ is $4A$. Therefore, the resistance is $\frac{1}{4} = \mathbf{0.25\Omega}$.

(ii) Does the resistance change above $1V$? Explain your answer.

Answer

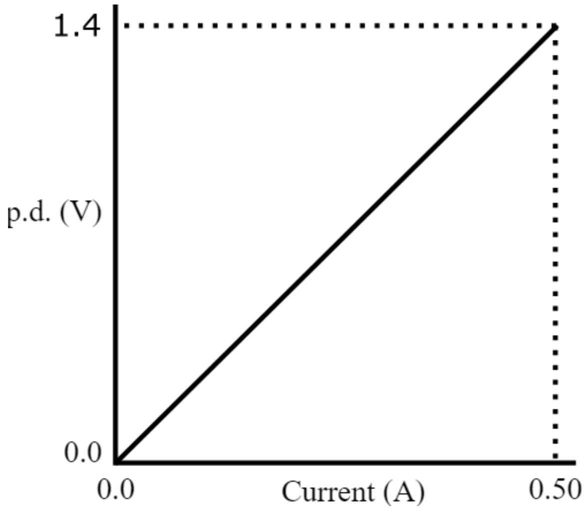
Yes, although the line has a constant gradient it doesn't start at the origin, so $R = \frac{V}{I}$ changes. The differential resistance, which is equal to the gradient of the line, is constant above $1V$. This is not the same as the resistance, which is given by $R = \frac{V}{I}$.

(iii) An ohmic device is one where the resistance does not depend on voltage for fixed physical conditions (e.g. constant temperature). What two features would you expect to see in a characteristic graph of an ohmic device?

Answer

The line starts at the origin and has a constant gradient.

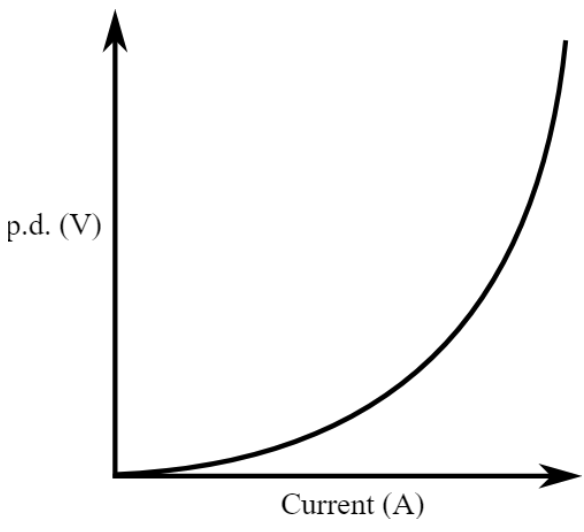
(iv) The following graph shows how the potential difference across a conductor varies with the current through it. Calculate the resistance of the conductor.



Answer

The resistance of the conductor is $\frac{1.4}{0.5} = \mathbf{2.8\Omega.}$

(v) The graph below shows how the potential difference across a conductor varies with the current through it. How does the resistance of the conductor change with increasing current?



Answer

As the slope of the slope of the graph increases, the resistance increases with increasing current.

5.6 Power Calculations

(i) How much current does a $2.0kW$ electric fire draw from the $230V$ mains?

Answer

Power is given by VI. Therefore the current is $\frac{2000}{230} = 8.69A$.

(ii) What is the power rating of a lamp which draws $0.26A$ from the $230V$ mains?

Answer

The power rating is $230 \times 0.26 = 59.8W$.

(iii) A torch bulb has $2.5V$, 0.18A stamped on it. What is its power rating?

Answer

The power rating is $2.5 \times 0.18 = 0.45W$.

(iv) What is the potential difference across a heater which develops power of $42W$ when a current of $3.5A$ flows through it?

Answer

The potential difference is $\frac{42}{3.5} = 12V$.

(v) The power of the heater element of a toilet hand dryer is $2100W$. It operates from the $230V$ mains. Calculate the current drawn from the mains.

Answer

The current drawn is $\frac{2100}{230} = 9.13A$

(vi) What current is carried in the element of a $2.4kW$ kettle connected to the 230V mains?

Answer

The current drawn is $\frac{2400}{230} = 10.43A$

(vii) An MRI scanner has a peak power of $35.0kW$. It is connected to a power supply at $415V$.What is the peak current drawn by the MRI scanner?

Answer

The current drawn is $\frac{35000}{415} = 84.33A$

(viii) On building sites, $115V$ mains is used to reduce the risk of harm by electric shock. A drill made for normal household (230V) use requires a current of $5.60A$. The manufacturer makes a model of similar power rating for use on building sites. What current will the builders' version need?

Answer

To get the same power for half the voltage, the current must be two times the original amount. Therefore, the builder's version needs a current of $11.2A$.

5.7 Resistance and Power

(i) What voltage is needed if $3.0W$ of power is going to be dissipated in a 4.5Ω resistor?

Answer

Power is given by $VI = I^2R$. The current is $\sqrt{\frac{3}{4.5}}A$. The voltage is $\sqrt{\frac{3}{4.5}} \times 4.5 = 3.67V$.

(ii) Some of the power coming into houses is wasted by the wires carrying the current due to their resistance. For a particular house, the wire which supplies it has a resistance of 1.5Ω. If the current is $83A$, what is the power wastage in the supply wire? For a different house, the rules say that no more than $6.0V$ may be 'dropped' across the supply wire. What power wastage does this correspond to if the wire has a resistance of 2.5Ω?

Answer

The power wastage is $83^2 \times 1.5 = 10333.5W$. In the second case, the current is $\frac{6}{2.5}A$. The power wastage is $\frac{36}{2.5} = 14.4W$.

(iii) What is the resistance of a $1.2kW$ light bulb operating on a voltage of $115V$?

Answer

The resistance is $\frac{115^2}{1200} = 11.02\Omega$.

(iv) Old lamp dimmers were variable resistors wired in series with the light bulb. Suppose you put a 25Ω resistor in series with a lamp such that the voltage across the lamp is only half of the $230V$ supply voltage.What is the power dissipated by the resistor?

Answer

The voltage across the resistor is $115V$. The power dissipated by the resistor is $\frac{115^2}{25} = 529W$.

(v) The National Grid operates at voltages of up to $400kV$. A 6.5Ω wire on the $400kV$ system is enabling $68MW$ of electrical power to be delivered to the customer. Calculate the current in the wire. Calculate the voltage 'dropped' along the wire.Calculate the power 'wasted' in the wire.

Answer

The current in the wire is $\frac{68000000}{400000} = \mathbf{170A}$. The voltage dropped across the wire is $\mathbf{170 \times 6.5 = 1105V}$. The power wasted is $\mathbf{170^2 \times 6.5 = 187850W}$.

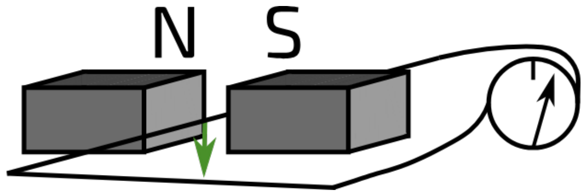
(vi) A resistor carries a current of $\mathbf{2.0A}$.If its resistance is $\mathbf{50\Omega}$, what is the power developed in it? If its resistance is 48Ω, what is the power developed in it?

Answer

The power developed in the 50Ω resistor is $\mathbf{2^2 \times 50 = 200W}$. The power developed in the 48Ω resistor is $\mathbf{2^2 \times 48 = 192W}$.

5.8 E-M Induction and Generators

(i) A long wire, connected to a centre-zero galvanometer, is moved downwards perpendicular to a magnetic field. The field is between two permanent magnets, with opposite poles facing each other as shown in the figure. While the wire is moving, the galvanometer needle moves to the right.



How would the pointer of the galvanometer move if the wire was moved up through the magnetic field? What would be the induced current in the conductor if it was held stationary in the centre of the magnetic field? State three ways of increasing the deflection of the pointer on the galvanometer when the wire is moved through the magnetic field.

Answer

The pointer would move to the left. The induced current in the conductor would be $0A$. Move the wire more quickly, use stronger magnets, or coil the wire up and move several turns of the wire between the magnets.

(ii) A pupil is investigating the effects of electromagnetic induction. He moves a conducting wire up and down perpendicular to the the straight line between two permanent magnets, expecting the pointer on the galvanometer to show a deflection. However, the pointer does not move. The meter is working and is sensitive enough for the experiment. There are no breaks in the wires. What must be wrong?

Answer

The magnets' poles facing each other are the same.

(iii) Suppose $+1.5mV$ is induced when a wire is moved between the poles of a large permanent magnet at a speed of $0.20\frac{m}{s}$ from left to right. What voltage would you expect when: the speed is increased to 0.40m/s? The speed is 0.3m/s, but the wire is moved from right to left? The original experiment is repeated with a magnet four times as strong?

Answer

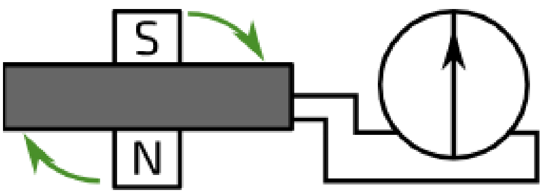
The expected voltage is $2 \times 1.5 = 3mV$. The expected voltage when the wire is moved from right to left is $-1.5 \times 1.5 = -2.25mV$. If the experiment is repeated with a magnet four times as strong, the expected voltage is $4 \times 1.5 = 6mV$.

(iv) Suppose $+2.7V$ is induced when the North pole of a magnet is inserted into a 200-turn coil at a speed of $1.5\frac{m}{s}$ from above. What voltage would you expect if: the coil had 450 turns on it? The magnet were moved into the 200-turn coil at $8.4\frac{m}{s}$? The South pole of the magnet were moved into the 200-turn coil at $4.5\frac{m}{s}$?If the North pole of magnet with $1000 \times$ the strength was held still inside the coil?

Answer

The expected voltage is $2.7 \times \left(\frac{450}{200}\right) = 6.075V$. The expected voltage when the magnet moves at $8.4\frac{m}{s}$ is $2.7 \times \left(\frac{8.4}{1.5}\right) = 15.12V$. When the south pole of the magnet moves into the coil, the sign of the voltage changes. Therefore the expected voltage if the South pole of the magnet were moved into the 200-turn coil at $4.5\frac{m}{s}$ is $2.7 \times \left(-\frac{4.5}{1.5}\right) = -8.1V$. There will be no voltage induced if there is no change in the magnetic field.

(v) A simple electrical generator is made by mounting a magnet in a coil of wire (shaded) so that it can be turned as in the diagram. It is connected to a meter which can read positive and negative voltages. One quarter of a turn later from the position shown on the diagram, the meter shows a positive voltage of $1.5V$.



(i) What will the meter show one half of a turn later still? (i.e. three quarters of a turn on from the position shown in the diagram?)

Answer

Half a turn later, the positions of the north and south poles of the magnet are interchanged, so the voltage will be $-1.5V$.

(ii) What will the meter show when the magnet is back in the position shown in the figure?

Answer

When the magnet is back in the position shown in the figure, the voltage will be $0V$.

(iii) What two things would happen if you turned the magnet twice as quickly?

Answer

The maximum reading would be greater and the reading would alternate between positive and negative twice as quickly

(iv) How could you modify the design so that larger voltages were induced without needing to change the speed of the rotation?

Answer

Have a stronger magnet and more coils in the wire

5.9 Transformers

(i) A doorbell for a house works from $8.0V$ a.c. To operate the bell from the $240V$ mains supply, a transformer can be used.How many turns would be in the primary winding for each turn in the secondary winding? Would the transformer be a step-up or a step-down type?

Answer

There would be 30 turns of primary winding for each turn in the secondary winding. The transformer is a step-down transformer.

(ii) To produce an output of $48V$ a.c. from an input of $240V$a.c., how many turns would be required in the primary winding if there were 100turns in the secondary?

Answer

The ratio of voltages is equal to the ratio of turns i.e. $\frac{240}{48} = \frac{P}{100}$ where P is the number of turns in the primary. Therefore the number of turns required in the primary winding is 500.

(iii) The input voltage to a step-down transformer is $240V$ a.c. at a frequency of $50Hz$. The primary winding has 6000turns and the secondary 300 turns.What is the voltage output? What is its output frequency?

Answer

The voltage output is $\frac{240}{20} = 12V$. The output frequency doesnt change, it is still $50Hz$.

(iv) A 12volt car battery is placed across the primary coil of a $1 : 20$ step-up transformer. What is the output voltage across the secondary?

Answer

Transformers only work for alternating current. The $12V$ car battery provides direct current so there is no change in magnetic field in the primary coil to induce a change in the voltage of the secondary. Therefore the output voltage across the secondary is $0V$.

(v) Calculate the current needed to carry the $2000MW$ generated at a large power station if the distribution voltage is kept at $22kV$, the generator output voltage.

Answer

The current is $\frac{2000 \times 10^6}{22000} = 91kA$.

(i) Calculate the current needed to carry $2000MW$ if the voltage is $400kV$.

Answer

The current is $\frac{2000\times10^6}{400000} = 5kA$.

(ii) The cables used have a total resistance of 9.0Ω. Calculate the power which would be wasted in heating the wire if the current of part (a) actually flowed in these cables.

Answer

The power wasted is $\frac{2000\times10^6}{22000} \times \frac{2000\times10^6}{22000} \times 9 \approx 74000MW$.

(iii) Calculate the power wasted if the current of part (b) passes through the wire.

Answer

The power wasted is $\frac{2000\times10^6}{400000} \times \frac{2000\times10^6}{400000} \times 9 \approx 225MW$.

(vi) A computer power supply unit can be switched to work on European ($230V$ a.c.) or United States ($115V$ a.c.) mains. In the European setting, there are 2000 turns on the primary coil. Assume that the secondary coil does not change.When switched into United States mode, how many turns are there now on the primary?

Answer

When switched into the United States mode the number of turns on the primary is $\frac{2000}{2} = 1000$.

6 Energy

6.1 Thermal Energy

(i) What is the change in thermal energy of 1.00kg of water that is raised from $20.0°C$ to its boiling point? (Assume the system is well insulated.)

Answer

We have $\Delta Q = mc\Delta T$. Here $\Delta T = 100 - 20 = 80°C$ and $m = 1kg$. Therefore, the change in thermal energy is
$\Delta Q = 1 \times 4200 \times 80 = 336000$J.

(ii) How much energy is required to raise the temperature of a 200g block of ice from -10.0°C? (The specific heat capacity of ice is 2100J/(kg°C). Assume the system is well insulated.

Answer

We have $\Delta Q = mc\Delta T$. Here $\Delta T = 0 - (-10) = 10°C$ and $m = 0.2kg$. Therefore, the change in thermal energy is
$\Delta Q = 0.2 \times 2100 \times 10 = 4200$J.

(iii) A 1.8kg block of ice, removed from a freezer at a temperature of -18°C, is placed in a fridge which has a temperature of 0.0°C. After a few hours, the ice has warmed up to the fridge temperature. What is the change in the stored thermal energy store of the block?

Answer

We have $\Delta Q = mc\Delta T$. Here $\Delta T = 0 - (-18) = 18°C$ and $m = 1.8kg$. Therefore, the change in thermal energy is
$\Delta Q = 1.8 \times 2100 \times 18 \approx 68$kJ.

(iv) Heating a room The specific heat capacity of air is 1000 $J/kg°C$.

a. How much energy would be needed to raise the temperature of the air in a room by 5.0°C, if the room measures $4.0m \times 4.0m \times 3.0m$? (Density of air=1.0kg/$m^3$). Assume that the room has no furniture and that the walls gain no thermal energy.

Answer

The mass of the air in the room is
$m = volume \times density = 4 \times 4 \times 3 \times 1 = 48kg$. $\Delta T = 5$°C.
Therefore $\Delta Q = mc\Delta T = 48 \times 1000 \times 5 = 240$kJ.

b. How long would a 1.0kW convection heater take to heat the air?

Answer

The energy required to heat the air is 240kJ. The power of the heater is $1kW$. Therefore, the time taken to heat the room is $\frac{240}{1} = 240s$.

(v) A carpet cleaning machine holds 40 litres of water. It is filled with water at 15 °C and the water is heated by a 3.0kW element to a temperature of70 ° C.

a. How much energy is added to the water?

Answer

The mass of the water is 40 kgs. The energy added to the water is $40 \times 4200 \times (70 - 15) = 9240$kJ.

b. Assuming there is no change in the thermal energy store in the machine or air, what is the time the heater takes to transfer the energy to the water?

Answer

The time to heat the water is given by $\frac{9240}{3} = 3080s$.

(vi) 75600J of thermal energy is needed to raise the temperature of a 2.0kg block of ice removed from a freezer at -18°C to its melting point. Calculate the specific heat capacity of ice suggested by these figures.

Answer

The specific heat capacity of ice is $\frac{75600}{2 \times 18} = 2100J/kg^{\circ}C$.

(vii) Calculate the specific heat capacity of a 3.0kg piece of metal which experiences a temperature rise of 25 °C when it is heated at a rate of 60W for 10minutes, if a total of 3000J heats the surroundings during this process.

Answer

The total energy from the heater is $60 \times 600 = 36000$J. The energy transmitted to the metal is $36000 - 3000 = 33000$J. Therefore the specific heat capacity of the metal is $\frac{33000}{3 \times 25} = 440J/kg^{\circ}C$.

(viii) Assuming the system is well insulated, what temperature would 490g of water reach, starting at 15 ° C, if a 60W heater heated it for 20 minutes?

Answer

Let T be the final temperature of water. The energy supplied to the water is $60 \times 1200s = 72$kJ which is equal to the change in the thermal engergy $0.49 \times 4.2 \times (T - 15)$kJ. Therefore, the final temperature is $15 + \frac{72}{0.49 \times 4.2} \approx 50$°C.

(ix) Calculate the thermal energy needed to raise the temperature of a 2.5kg block of ice to its melting point, if it is taken from a freezer at -18°C.

Answer

The thermal energy needed is $2.5 \times 2.1 \times (0 - (-18)) \approx 95$kJ.

(x) How much thermal energy is gained when the temperature of 800g of iron is raised by 120°. The specific heat capacity of iron is 440J $/kg$°C.

Answer

The thermal energy gained is $0.8 \times 440 \times 120 \approx 42$kJ.

(xi) Copper has s specific heat capacity of 390 J/kg°C. A 20g piece of copper at 1050 ° is dropped into a very large tank of water which is at 15°C. What is the change in the thermal energy of the water when the copper has cooled to the temperature of the water?

Answer

The change in the thermal energy of water is
$0.02 \times 390 \times (1050 - 15) = 8073$J.

(xii) When 30g of gold is warmed to 50°C, the change in its thermal energy is 260J. Calculate the thermal capacity of gold.

Answer

We have $m = 0.03kg$, $\Delta T = 50$°C and $\Delta Q = 260$kJ. Therefore, the specific capacity of gold is $\frac{260}{0.03 \times 50} = 173.3J/kg^{\circ}C$.

(xiii) 10kg of water at 40°C is mixed with 10kg of water at 20°C in a bath. Assuming that the total energy stored in the water does not change (none is lost to the surroundings), what will the final temperature of the water be?

Answer

Let T be the final temperature of the water. The energy lost by the hot water should be equal to the energy gained by the cold water as no energy is lost to the surroundings. Therefore, we have
$10 \times c \times (40 - T) = 10 \times c \times (T - 20) \implies T = 30$°C.

6.2 Latent Heat

(i) A student measures 250g of water and pours it into a beaker. They heat the water over a Bunsen burner for five minutes, then measure the mass of the water again; this time it is 200g. The specific latent heat of vaporisation of water is 2260kJ/kg. How much energy has been transferred in evaporating the water?

Answer

We see that $250 - 200 = 50g$ of water has been vaporized. Therefore the amount of energy transferred in evaporating the water is $0.05 \times 2260 = 113$kJ.

(ii) Pure water boils at 100°C, has a specific heat capacity of 4200J/kg°C and a specific latent heat of vaporization of 2260kJ /kg.

a. How much energy is required for 2.0kg of water to boil if it is already 100 ° C?

Answer

The energy required to boil water already at $100°$ is $2 \times 2260 = 4520$ kJ.

b. How much energy is required if the water started at 40 °?

The energy required to get the water to 100 ° is $2 \times 4200 \times 60 = 504$ kJ. The amount of energy required to boil water at 100 ° is 4520kJ. Therefore the total energy required is $4520 + 504 \approx 5000$kJ.

(iii) 1000kg of steam is condensed back to water in the condenser of a power station each hour. The specific latent heat of vaporisation of water is 2260kJ/kg.Calculate the energy output to the environment this causes each second.

Answer

Amount of energy released from steam in $1hr$ is $1000 \times 2260 = 2260$ MJ. The energy output per second is $\frac{2260}{3600} \approx 0.63$MJ.

(iv) A typical fluid used in a fridge has a latent heat of vaporisation of 200kJ/kg. The fluid needs to remove 30J of thermal energy from the fridge each second, and it does this by boiling alone. Calculate the minimum mass of fluid which must flow through the fridge each second.

Answer

Let m_f be the mass of fluid in the fridge. The energy removed by the fluid every second is $m_f \times 200$kJ which is equal to 0.03kJ. Therefore the mass of the fluid is $m_f = \frac{0.03}{200} kg = 0.15$g.

(v) How much energy would be required to enable 5.0×10^{-3} kg of ethanol to evaporate? The specific latent heat of vaporisation of ethanol is 840kJ/kg.

Answer

The amount of energy required is $5.0 \times 10^{-3} \times 840 = 4.2$kJ.

(vi) A sample of solid ethanoic acid is at its melting point of 17.0 °C. It has a specific latent heat of fusion of 192000J/kg. How much ethanoic acid can be melted with 864kJ of thermal energy?

Answer

The amount of ethanoic acid that can be melted is $\frac{864}{192} = 4.5$kg.

(vii) Liquid nitrogen boils at minus, −196 °C.40.0kg of liquid nitrogen in a dewar flask completely evaporates when 7.96MJ of thermal energy is transferred. What is its latent heat of vaporisation?

Answer

The latent heat of vaporization is $\frac{7960}{40} = 199$kJ/kg.

(viii) A 1000W heater is placed in an insulated beaker containing 750g of water at 100 ^circC. The water vapour is allowed to escape. Assume that there is no loss to the surroundings via conduction, convection or radiation. The specific latent heat of vaporisation of water is 2260kJ/kg.

a. How much water is left after 5.0 minutes?

   Answer

   The total amount of heat transmitted to the water in 5 minutes is $1000 \times 5 \times 60 = 300$kJ. The amount of water that evaporated is $\frac{300}{2260} = 0.13$kg. The amount of water left is $750 - 130 = 620$g.

b. How long will it take for half of the water to have evaporated? Give your answer to the nearest second.

   Answer

   The amount of energy required to evaporate half of the water i.e. 375 gm is $0.375 \times 2260 = 847.5$kJ. The amount of time the heater takes to provide 847.5kJ is 848s.

6.3 Payback Times

(i) Two of the domestic upgrades with highest installation costs are solar panels and double glazed windows, whereas cavity wall insulation is more midrange.A photovoltaic solar panel costs $£5000$ to install and saves $£100 per year$ in electricity costs. What is the payback time in years?

Answer

The payback time in years is $\frac{5000}{100} = 50$ years.

(ii) Fitting an entire house with double glazed windows costs $£9000$. Once fitted, the windows provide a saving of $£800 each year$ in the heating bill. What is the payback time in months?

Answer

The payback time in years is $(\frac{9000}{800})12 = 135$ months.

(iii) Fitting cavity wall insulation to a flat costs $£400$. The annual saving in heating bills is $£90$. What is the payback time in months?

Answer

The payback time in months is $\frac{400 \times 12}{90} = 53$ months.

(iv) A jacket for a hot water tank costs $£30$ and the payback time is 8.0 months. How much money does using the jacket save each year? Give your answer as an integer number of pounds.

Answer

The amount of money the jacket saves each year is $(\frac{30}{8})12 = £45.$

(v) Photovoltaic solar panels cost $£500$per square metre to install on a $6.0m \times 6.0m$ roof. They save the owner $£75$per month. What is the payback time? Give your answer as an integer number of years.

Answer

The total cost of the installation of the solar panels is $500 \times 6 \times 6 = £18000.$ The payback time in years is $\frac{18000}{75 \times 12} = 20$ years.

(vi) A builder quotes that they can fit loft insulation to your house for $£300$ and it will pay for itself in two years.How much money will you save in heating bills over ten years if their quote is accurate? Give your answer as a whole number of pounds.

Answer

The amount of money saved each year is $\frac{300}{2} = £150.$ The amount of money saved in 10 years is $150 \times 10 = £1500.$

(i) What is the profit after ten years (money saved in heating minus the installation cost) if you follow their advice? Give your answer as a whole number of pounds.

Answer

The profit after 10 years = $1500 - 300 = £1200.$

(vii) A photovoltaic cell costs just $£3000$ to install but saves only $£50$ per year in electricity costs.What is the payback time? Give your answer as an integer number of years.

Answer

The payback time is $\frac{3000}{50} = 60$ years.

(viii) A wind turbine can generate $150W$ on average and costs $£3000$ to buy and install. What is the necessary cost per kWh if the owner wanted to pay for the turbine with one year's electricity sales? Give your answer as a number of pence to the nearest penny.

Answer

Let the cost per kWh be $£c.$ The electricity sales for the entire year is $365 \times 24 \times 0.150 \times c = £1314c = £3000.$ Therefore the cost per kWh is $c = \frac{3000}{1314} = £2.28$

(i) What is the cost per kWh if the owner wants to pay for the turbine with five year's electricity sales? Give your answer as a number of pence to the nearest penny.

Answer

The cost per kWh is $\frac{3000}{1314 \times 5} = £0.46.$

(ii) If the turbine owner can sell at $15p$ per kWh, what is the payback time? Give your answer as a number of years, to the nearest year.

Answer

The amount of sales in 1 year is $1314 \times 0.15 = £197.1.$ Therefor the payback time in years is $\frac{3000}{197.1} = 15.$

(ix) A small photovoltaic phone charger is able to charge a mobile phone at a voltage of $5.00V$ and a current of $2.00A$. The charger can only operate between the hours of 8am and 7pm. The charger costs $£40.15$ and saves the user $10.0p$ perkWh when used.What is the payback time? Give your answer as a number of years, to the nearest year.

Answer

Total energy provided by the charger in a year is $365 \times 11 \times 5 \times 2 = 40.150kWh$. The amount of money saved in a year is $40.15 \times 0.1 = £4.015$. The payback time in years is $\frac{40.15}{4.015} = 10$.

(x) A caravan owner used to pay $15p$ per kWh for the electricity to power his $35W$ fridge. He bought a wind turbine to generate the electricity instead. In seven years time, he will have saved enough money on his bills to pay for the turbine. How much did the turbine cost? How much did the turbine cost? Give your answer as a number of pounds to the nearest whole number of pounds.

Answer

The electricity cost of the fridge per year is $365 \times 24 \times 35 \times \frac{0.15}{1000} = £45.99$. The amount of money saved on bills in 7 years is $£321.93$. The cost of the turbine is $£322$.

(xi) A parent used to leave a $2.0W$ nightlight running continuously. They then buy a timer so that the light is only on from 7pm to 7am, but it doesn't make a difference to the electricity usage. What is the power used by the timer itself? Give your answer in watts.

Answer

The amount of energy consumed by the nightlight in a day is $48Wh$. Let the power of the timer be pW. The timer runs for 24 hours in a day while the bulb only runs for 12 hours. The amount of energy consumed when the timer is on is $24p + 2 \times 12 = 48Wh$. as there is no difference in electricity usage. The power used by the timer is $p = \frac{48-24}{24} = 1W$.

6.4 Doing Work, Potential Energy and Power

(i) A builder needs to drag a sack of cement 20m along the floor against a friction force of 60N.

a. Calculate the work done.

Answer

The amount of work done is $60 \times 20 = 1200$J.

b. If the builder took two minutes to do the dragging, what was their power?

Answer

The power of the builder is $\frac{1200}{120} = 10$W.

(ii) How much gravitational potential energy is lost when a 60kg boy walks down a flight of stairs which is 4.5m high?

Answer

The amount of gravitational potential energy lost is $mgh = 60 \times 10 \times 4.52700$J.

(iii) A weight-lifter raises a barbell of mass 20kg, doing 490J of work on it. Through what height does she lift the barbell?

Answer

The height is $\frac{490}{20 \times 10} = 2.45$m.

(iv) A lighting bar on stage has a mass of 300kg when supporting stage lights.

a. What is its weight?

Answer

The weight of the bar is $300 \times 10 = 3000$N.

b. How much energy do you need to lift it by 10, m,10m?

Answer

The energy need to lift the bar by 10m is $3000 \times 10 = 30000$J.

c. If your power is 100W, how long would it take you to lift the bar by 10m?

Answer

The time taken to lift the bar by $10m$ is $\frac{30000}{100} = 300s$.

d. What is the increase in gravitational potential energy when the bar is lifted by 10m?

Answer

The increase in gravitational potential energy is
$300 \times 10 \times 10 = 30000$J.

(v) A car park has three floors. The ground floor is at street level. The first floor is 4.0m above the ground floor, the second floor is 3.0m above the first floor.

a. How much energy does it take to lift an 800kg hatchback to the top level from the street?

Answer

The energy required is equal to the gravitational potential energy = $mgh = 800 \times 10 \times 7 = 56000$J.

b. How much energy does it take to lift a 2000kg SUV to the first floor from the street?

Answer

The energy required is equal to the gravitational potential energy = $mgh = 2000 \times 10 \times 4 = 80000$J.

c. Calculate the change in gravitational potential energy when a 400kg city car moves from the street to the top floor.

Answer

The energy required is equal to the gravitational potential energy = $mgh = 400 \times 10 \times 7 = 28000$J.

d. Calculate the change in gravitational potential (that is, the gravitational potential energy per kilogram) for the hatchback when it is lifted to the top level from the street.

Answer

The gravitational potential is $\frac{56000}{800} = 70J/kg$.

e. Calculate the change in gravitational potential (that is, the gravitational potential energy per kilogram) for the SUV when it is lifted to the first floor from the street.

Answer

The gravitational potential is $\frac{80000}{2000} = 40J/kg$.

f. Calculate the change in gravitational potential (that is, the gravitational potential energy per kilogram) for the city car when it is lifted to the top floor from the street.

Answer

The gravitational potential is $\frac{28000}{400} = 70J/kg$.

(vi) When an object falls in a gravitational field it loses gravitational potential energy.

(i) How much gravitational potential energy is lost when a rock of mass 3.0kg falls to the foot of a 250m cliff on the Moon where 'g' is 1.6N/kg?

Answer

The gravitational energy is $mgh = 3.0 \times 1.6 \times 250 = 1200$J.

(ii) In a hydroelectric power station, how much potential energy is lost by 100 tonnes of water flowing down through the pipes, falling a vertical distance of 200metres? [1 tonne=1000kg.]

(vii) A rower exercises with a power of 200W. Her boat (containing 7 other equally-strong rowers) travels at a speed of 9.0m/s.

(i) How far will the boat go in 20s?

Answer

The boat will go $20 \times 9 = 180$m.

(ii) How much energy will the crew have converted in 20s?

Answer

The energy converted in $20s$ is $200 \times 20 \times 8 = 32000$J.

(iii) Calculate the force each rower exerts to push the boat along.

Answer

Let the force each rower exerts is FN. We have
$F \times 9 = 200 \Longrightarrow F = 22.2$N.

(iv) What happens if you multiply the force in (c) by the speed?

Answer

You get the power of one rower as $Power = Force \times Speed$.

(viii) A car is being driven down a drag race track.

(i) Calculate the power of car engine needed if the car is to be driven at a constant 70mph (31m/s) against a combined friction and air resistance force of 1400N.

Answer

The power needed is $\mathbf{1400 \times 31 = 43400}$m/s.

(ii) Calculate the power of car engine needed if the car is to be driven at a constant 100mph (44.7m/s) against a combined friction and air resistance force of 1400N.

Answer

The power needed is $\mathbf{1400 \times 44.7 = 62580}$m/s.

(ix) A mountain climber has a mass (with all of their equipment) of 95.0kg.If they were perfectly efficient, and ate 500g of chocolate (providing 11.1MJ of chemical energy), how high could they climb?

Answer

The amount of energy provided by the chocolate should be equal to the amount of gravitational potential energy gained by the climber. If the climber climbs a height hm, the gravitational energy is $\mathbf{95 \times 10 \times \boldsymbol{h} = 11.1 \times 10^{6}}$. Therefore the climber can climb $\mathbf{\frac{11100000}{950} = 11.7}$km.

6.5 Kinetic Energy

(i) Calculate the kinetic energy of a 2.0kg motion trolley going at $3.0\frac{m}{s}$.

Answer

The kinetic energy of the trolley is $\frac{1}{2} \times 2 \times 3^2 = 9J$.

(ii) A $300000kg$ airliner is flying at $250\frac{m}{s}$ at an altitude of $11000m$. How large is its kinetic energy when expressed as a percentage of the total kinetic and gravitational potential energy?

Answer

The total potential energy of the airliner is $mgh = 300000 \times 10 \times 11000 = 33 \times 10^9 J$. The kinetic energy of the airlines is $\frac{1}{2} \times 300000 \times 250 \times 250 = 9375000000 = 9.375 \times 10^9 J$. Therefore kinetic energy as a percentage of the total energy is $\left(\frac{9.375}{33+9.375}\right) \times 100 = 22.1\%$.

(iii) How fast was a 1400kg car travelling if it lost $280kJ$ of kinetic energy in coming to a stop?

Answer

As the car came to a stop, it lost all of its kinetic energy. Let the speed of the car be $v\frac{m}{s}$. The kinetic energy of the car is $\frac{1}{2} \times 1400 \times v^2 = 280000 \Rightarrow v = 20\frac{m}{s}$.

(iv) A car of mass $1200kg$ slows down from a speed of $20\frac{m}{s}$ to $10\frac{m}{s}$. How much kinetic energy does the car lose? (Hint: first work out the kinetic energy before and after the deceleration.)

Answer

The kinetic energy lost by the car is $\frac{1}{2} \times 1200 \times (20^2 - 10^2) = 180kJ$.

(v) What is the mass of an object travelling at $8.0\frac{m}{s}$ which has $96J$ of kinetic energy?

Answer

Let the mass of the object be m kg.We have $\frac{1}{2} \times m \times 8^2 = 96 \Rightarrow m = 3kg$.

(vi) At what speed is a $250g$ stone moving if its kinetic energy is 3.5joules?

Answer

Let the speed of the stone be v kg.We have
$\frac{1}{2} \times 0.25 \times v^2 = 3.5 \Rightarrow v = 5.3\frac{m}{s}.$

(vii) A $500kg$ pumpkin is dropped $15m$ on top of a school bus.

(i) How much gravitational potential energy was gained when it was winched up $15m$?

Answer

The gravitational potential energy gained by the pumpkin is
$mgh = 500 \times 10 \times 15 = 75kJ.$

(ii) Assuming all of this GPE is turned into kinetic energy as the pumpkin drops, work out its speed as it hits the school bus.

Answer

Let the speed when the pumpkin hits the bus be $v\frac{m}{s}$. As energy is conserved, we have $\frac{1}{2} \times 500 \times v^2 = 75000 \Rightarrow v = 17.3\frac{m}{s}.$

(iii) Would the speed be any different for a $5.0kg$ pumpkin?

Answer

No, it is the same. In both cases, the initial speed of the pumpkin is the same and the only acceleration is due to gravity.

(viii) A $700kg$ car accelerates uniformly from rest to $30\frac{m}{s}$ in $10s$. Calculate its acceleration.

Answer

We have $v = u + at.$ as the acceleration is uniform. Here $u = 0\frac{m}{s}$ and $v = 30\frac{m}{s}$ and $t = 10s$. The acceleration is $\frac{30}{10} = 3\frac{m}{s^2}.$

(i) Calculate the force needed to give the car this acceleration.

Answer

The force needed for the accleration is $m \times a = 700 \times 3 = 2100N.$

(ii) The kinetic energy equals the work done in accelerating the car. Use this fact to calculate the kinetic energy.

Answer

The distance travelled by the car is given by
$s = ut + \frac{1}{2}at^2 = \frac{1}{2} \times 3 \times 10^2 = 150m.$ The work done in moving the car $150m$ with a force of $2100N$ is $315kJ.$

6.6 Efficiency

(i) A battery powered toy car accelerates from $0.0\frac{m}{s}$ to $8.5\frac{m}{s}$. Its mass is $0.55kg$. The chemical store in its battery is decreased by $30J$. Calculate the efficiency of this process.

Answer

The kinetic energy gained by the car is $\frac{1}{2} \times 0.55 \times 8.5^2 = 19.87J$. The decrease in chemical energy in the battery is $30J$. The efficiency is $\left(\frac{19.87}{30}\right)100 = 66\%$.

(ii) A mains operated motor raises an $8500N$ weight to a height of $2.7m$. The electrical work done by the mains supply is $29000J$.Calculate the efficiency of this process.

Answer

The work done on the weight is $8500 \times 2.7 = 22950J$. The work expended by the mains supply is $29000J$. The efficiency is $\left(\frac{22950}{29000}\right)100 = 79\%$.

(iii) A mains powered electric winch pulls a trolley up a ramp, raising it by a vertical distance of $1.2m$. The trolley's weight is $6200N$. The electrical work done by the mains supply is $9350J$.

Answer

The work done on the weight is $6200 \times 1.2 = 7440$. The work expended by the mains supply is $9350J$. The efficiency is $\left(\frac{7440}{9350}\right)100 = 79.6\%$.

(iv) A student plugs her phone in for an hour to charge the battery. The power supply does $11000J$ of electrical work (at a steady rate) and the chemical energy stored in her phone battery increases by $8300J$ (also at a steady rate).

(i) Calculate the efficiency of this process.

Answer

The efficiency of the process is $\left(\frac{8300}{11000}\right)100 = 75.4\%$

(ii) Calculate the increase in thermal energy resulting from 1.0hour of charging her phone.

Answer

As total energy is conserved, the amount of thermal energy is the difference between the electrical energy and chemical energy = $11000 - 8300 = 2700J$.

(i) Calculate the increase in the thermal energy in the first minute.

Answer

The thermal energy increases to $2700J$ in 60 mins. The increase in thermal energy in the first minute is $\frac{2700}{60} = 45J$.

(ii) In the first minute of charging, it is reasonable to assume that all of the increase in the thermal energy store raises the temperature of her phone battery. Her battery has a mass of $28g$ and its specific heat capacity is $480\frac{J}{kg^{\circ}C}$.Calculate the battery's temperature rise during the first minute.

Answer

Energy required in raising the battery temperature by $T^{\circ}C$ is $0.028 \times 480 \times T = 45 \Rightarrow T = 3.3^{\circ}C$.

(v) A mains transformer has an input power of $2.0kW$ and is 90% efficient. How much energy would be wasted in 10minutes?

Answer

The power wasted is $2.0 \times 10\% = 0.2kW$. The energy wasted in 10 mins is $0.2 \times 10 \times 60 = 120kJ$.

(vi) A machine has an efficiency of 60%, the useful power output is $150W$. What is the total input power?

Answer

The total input power is $\frac{100 \times 150}{60} = 250W$.

(vii) An electric motor has a power input of 10watts when lifting a weight with a pulley system. The motor and pulley system is 80% efficient. Calculate how much potential energy would be gained by the weight in $5.0s$.

Answer

The amount of useful power i.e. power used to lift the weight is $10 \times 0.8 = 8W$. The potential energy gained by the weight in $5s$ is $8 \times 5 = 40J$.

(viii) An electric motor has a power input of 3.0watts when lifting a weight. The weight gains 10joules of potential energy in 5.0seconds. What is the useful output power of the motor? What is the motor's efficiency in carrying out the operation?

Answer

The useful output power is $\frac{10}{5} = 2W$. The efficiency is $(\frac{2}{3})100 = 66.66\%$.

(ix) A model hydroelectric power station produces just enough electric power to light a $6.0W$ lamp. The model is found to be 80% efficient at converting the potential energy store of the water into electrical work. What is the input power of the water running through the pipes?

Answer

Let the input power be I W. Useful output power is $0.8I = 6 \Rightarrow I = 7.5W$.

(x) An electric motor draws $2.0A$ from a $12V$ supply. It can lift a weight so that the weight gains $54J$ of potential energy in $3.0s$.

(i) Calculate the input power of the motor.

Answer

The input power of the motor is $12 \times 2 = 24W$.

(ii) Calculate the useful output power of the motor.

Answer

The output power of the motor is $\frac{54}{3} = 18W$.

(iii) Calculate the efficiency of the motor.

Answer

The efficiency of the motor is $(\frac{18}{24})100 = 75\%$.

(xi) A water pump, rated at $12V$; $5.0A$ raises $30kg$ of water through a height of $2.0m$ in a time of 15seconds. Calculate the pump's efficiency. [Assume the water has no kinetic energy on reaching the top (and take g=10N/kg)].

Answer

The input power of the pump is $12 \times 5 = 60W$. The work done by the pump is equal to the potential energy gained by the water which is

$30 \times 10 \times 2 = 600J$. The output power of the pump is $\frac{600}{15} = 40W$. The efficiency of the pump is $\left(\frac{40}{60}\right)100 = 67\%$.

(xii) A hydroelectric power station generates $64MW$ of electric power when the input power from the falling water is $70MW$.Calculate the efficiency of the system.

Answer

The efficiency of the system is $\left(\frac{64}{70}\right)100 = 91\%$.

6.7 Power and the Human Body

(i) A weight-lifter lifts a mass of $20kg$ through $1.5m$, ten times in one minute.

  (i) What is the weight of the 20kg mass?

  Answer

  The weight of the $20kg$ mass is $20 \times 10 = 200N.$

  (ii) Calculate the total work done in lifting the weights.

  Answer

  The work done in lifting the weight once is $200 \times 1.5 = 300J.$ The total work done in one minute is $300 \times 10 = 3000J.$

  (iii) Calculate the weight-lifter's average power in watts.

  Answer

  The average power of the weight-lifter is $\frac{3000}{60} = 50W.$

(ii) A $70kg$ bricklayer needs to put 100bricks ($2.0kg$ each) on the first floor of some scaffolding. The first floor of the scaffolding is 3.0m above the ground floor.

  (i) Calculate the potential energy change when the bricks are lifted to the first floor.

  Answer

  The potential energy change is $2 \times 100 \times 10 \times 3 = 6000J.$

    (i) Assuming that they move five bricks at a time, calculate the energy needed only for the bricklayer to climb the ladder enough times, in order to lift 100 bricks to the first floor.

    Answer

    As a bricklayer can only carry 5bricks at a time, he or she would need to climb the ladder 20 times.The energy required to carry his own weight up the ladder to transport 100 bricks is $70 \times 10 \times 3 \times 20 = 42000J.$

  (ii) If the bricklayer's maximum power is $800W$, how much time would they spend going up the ladder while doing the jobs in parts (a) and (b) combined?

Answer

The total energy required is $42000 + 6000 = 48000J$. The time required to complete the job of carrying 100 bricks is $\frac{48000}{800} = 60$s.

(iii) The power needed to keep a human being alive is called the basal metabolic rate (BMR). For adults this is about $6.0MJ$/day.

(i) Calculate the BMR in watts (J/s).

Answer

A day has $24 \times 3600s$, therefore BMR in watts is $\frac{6000000}{24 \times 3600} = 69.4W$.

(ii) Calculate the BMR in joules per hour (J/h).

Answer

The BMR in Joules per hour is $\frac{6000000}{24} = 250000\frac{J}{h}$.

(iv) A cyclist on an exercise bike has a basal metabolic rate of $100W$. Her muscles are 30% efficient. This means that for every $100J$ of energy given to the muscles in the form of food, only $30J$are converted into work done on the bike. The power reading on the exercise bike is $150W$.Calculate the total power needed by her body to produce this output.

Answer

Let P be the total power needed by her body to produce an output of $150W$.. We have $(P - 100) \times 0.3 = 150 \Rightarrow P = 600W$.

(v) The chemical processes in your body generate thermal energy, which keeps you warm. If you lose $30J$ of thermal energy each second to your surroundings, your body needs to convert another $30J$ into thermal energy each second to maintain body temperature. If this doesn't happen, your body temperature will fall, and you may become ill. Fred's body has a surface area of $2.0m^2$. He loses $80J$ of thermal energy each second to his surroundings.

(i) What basal metabolic rate is needed (in J/s) for Fred to keep his body temperature constant?

Answer

As Fred is losing $80\frac{J}{s}$ to his surroundings, he needs to convert $80\frac{J}{s}$ into thermal energy to maintain a constant body temperature. Therefore, the BMR is $80\frac{J}{s}$.

(ii) How much thermal energy does Fred lose per second through each square metre of body surface?

Answer

Through each square meter, Fred loses $\frac{80}{2} = 40J$.

(iii) Fred's baby sister has a surface area of $0.20m^2$. How much thermal energy do you expect her to lose each second?

Answer

She is expected to lost $40 \times 0.2 = 8J$ each second.

(iv) Who will find it easier to stay warm - Fred or his baby sister?

Answer

His sister loses $\frac{1}{10}$ of the thermal energy per second that Fred loses as she has $\frac{1}{10}$ of the area. As such, it would be expected that his sister would find it easier to stay warm, as she would require a slower metabolic rate to keep a constant temperature. However, she has very little body mass compared to Fred, much less than $\frac{1}{10}$, so she finds it much harder to keep warm.

6.8 Springs and Elastic Deformation

(i) Two identical springs each have a spring constant of $200\frac{N}{m}$. A mass of $9.7kg$ is hung from one spring, which is supported by the other. The top spring is supported by a strong beam.

(i) What is the total extension of the springs?

Answer

The force on each spring is the same which is the weight of the mass. Let the extension of each spring be em. The force on one spring is $200 \times e = 9.7 \times 10 \Rightarrow e = 0.485m$. The total extension of the springs is $0.485 \times 2 = .97m$.

(ii) The two springs are now set up in parallel to share the load.What will the extension of each spring be when the mass is supported?

Answer

As the two spring share the load, the force on each spring is half the weight of the mass. Let e be the extension of a spring. The force on each spring is $200 \times e = \frac{9.7 \times 100}{2} \Rightarrow e = 0.2425.$

(ii) Calculate the elastic potential energy when a $1000\frac{N}{m}$ spring is stretched by 6.0cm from its natural length.

Answer

The elastic potential energy is $\frac{1}{2} \times 1000 \times (0.06)^2 = 1.8J$

(iii) Calculate the elastic potential energy when a 1000N/m spring is stretched by a $20N$ force.

Answer

The extension in the spring is $\frac{20}{1000} = 0.02m$. The elastic potential energy is $\frac{1}{2} \times 1000 \times (0.02)^2 = 0.2J$.

(iv) Calculate the extension of a $500\frac{N}{m}$ spring when storing $3.0J$.

Answer

Let the extension be em. We have $\frac{1}{2} \times 500 \times e^2 = 3 \Rightarrow e = 0.11m$.

(v) Calculate the elastic potential energy when a $40N$ force stretches a spring by $0.50cm$.

Answer

The elastic potential energy is $\frac{1}{2} \times 40 \times (0.5)^2 = 5J.$

(vi) How much strain energy is stored in a $40k\frac{N}{m}$ spring when it is stretched by $3.0cm$?

Answer

The total energy stored is $\frac{1}{2} \times 40000 \times (0.03)^2 = 18J.$

(i) The string (from part A) is now relaxed by reducing the extension to $1.5cm$. Calculate how much strain energy has been released.

Answer

The total energy remaining when the string is stretched by 1.5cm is $\frac{1}{2} \times 40000 \times (0.015)^2 = 4.5J.$ Therefore, the energy released is $18 - 4.5 = 13.5J.$

7 Waves and Optics

7.1 Wave Properties and Basic Equations

(i) A musical note has a frequency of $440Hz$. The speed of sound in air is $330\frac{m}{s}$.

  (i) What is the wavelength of the sound?

  Answer

  The wavelength of the sound is $\frac{330}{440} = 0.75m$.

  (ii) What is the time period of the sound?

  Answer

  The time period of the sound is $\frac{1}{440} = 0.0023s$.

(ii) An ultrasound pulse has a wavelength of $1.0mm$. Its speed in water is $1400\frac{m}{s}$.

  (i) What is the frequency?

  Answer

  The frequency is $\frac{1400}{0.001} = 1400000Hz$.

  (ii) What is the time period of the sound?

  Answer

  The timeperiod is $\frac{1}{1400000} = 7.1 \times 10^{-7}.s$

(iii) A lighthouse flashes once every $7.1s$. What is its frequency?

Answer

The frequency is $\frac{1}{7.1} = 0.14Hz$.

(iv) The mains power has a frequency of $50Hz$.

Answer

The timeperiod is $\frac{1}{50} = 0.02s$.

(v) When a musical note goes up one octave in pitch, its frequency doubles. What happens to its wavelength?

Answer

As the speed remains the same, the wavelength halves.

(vi) Calculate the frequency of a water wave which has a wavelength of $1.5m$ and travels a distance of 10metres in 5.0seconds.

Answer

The speed of the wave is $\frac{10}{5} = 2\frac{m}{s}$. The frequency of the wave is $\frac{2}{1.5} = 1.33Hz$.

(vii) A water wave, travelling at $2.5\frac{m}{s}$, has a wavelength of $50cm$. What is the period of the wave?

Answer

The frequency of the wave is $\frac{2.5}{0.5} = 5Hz$. The period of the wave is $\frac{1}{5} = 0.2s$.

(viii) On a stormy day, a girl counts the number of wave crests breaking on the shore and finds that there are 60 in 4.0minutes. Calculate the water waves' frequency in hertz (waves per second).

Answer

The frequency is $\frac{60}{4\times60} = 0.25Hz$.

(ix) A tuning fork makes the musical note one octave above 'middle C' (in the 'scientific designation'). What is its frequency, if its prongs vibrate 2560 times in 5.0s?

Answer

The frequency is $\frac{2560}{5} = 512Hz$.

(x) What is the wavelength of a $200Hz$ sound wave in the air if the speed of sound in air is $340\frac{m}{s}$?

Answer

The wavelength is $\frac{340}{200} = 1.7m$.

(xi) What is the speed of sound through an aluminium rod if a sound vibration of frequency $13kHz$ has a wavelength of $40cm$?

Answer

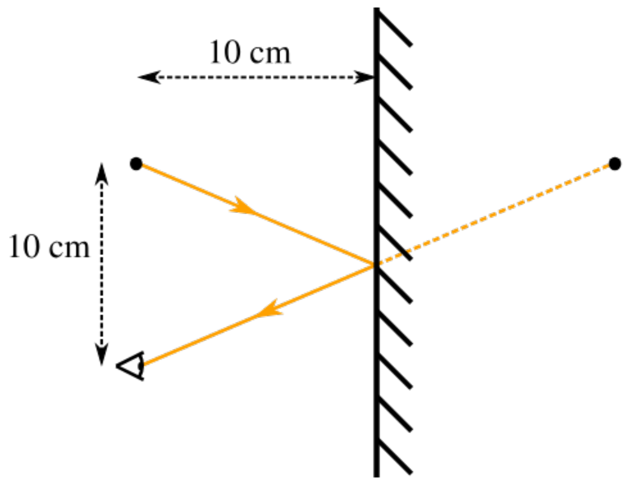
The speed of sound in aluminium is $13000 \times 0.4 = 5200\frac{m}{s}$.

7.2 Reflection - Plane Mirrors

(i) An object is $10cm$ away from a plane mirror. An observer is the same distance away from the mirror, and $10cm$ away from the object.

  (i) Sketch a ray diagram of the situation.

  Answer



  (ii) How far has the light travelled from the object to the observer via the plane mirror?

  Answer

  The distance the light travelled from the object to the observer via the mirror is given by $2\sqrt{5^2 + 10^2} = 22.36cm$

  (iii) What is the angle of incidence?

  Answer

  The angle of incidence is $\arctan(\frac{5}{10}) = 26.56^{\circ}$.

  (iv) How far away from the image is the observer?

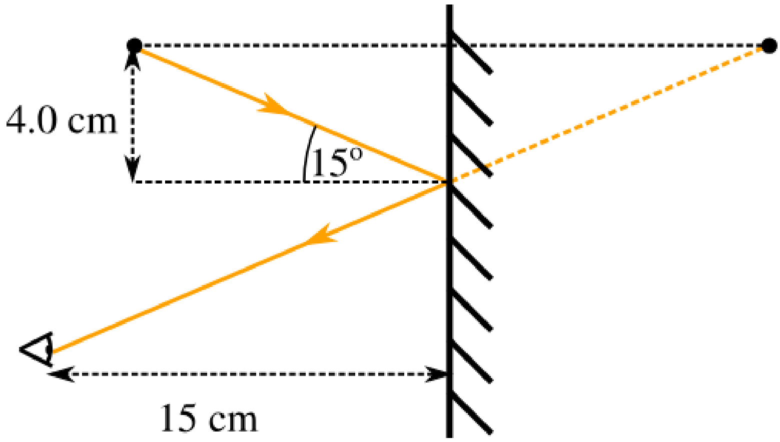
  Answer

  The image is $\sqrt{20^2 + 10^2} = 22.36cm$ from the observer.

(ii) An object is placed at a mystery distance from a plane mirror. An observer is $15cm$ away from the mirror. The angle of incidence is $15°$. The ray that travels from the object to the observer is incident on the plane mirror a perpendicular distance of 4.0cm from the imaginary line that connects the object to the

(i) Sketch a ray diagram of the situation.

Answer



(ii) What is the length of the ray from the object to the mirror?

Answer

The length of the ray from the object to the mirror is $\frac{4}{\sin(15^{\circ})} = 15.4cm.$

(iii) What is the distance between the object and the image?

Answer

The distance between the object and the image is $2 \times \frac{4}{\tan(15^{\circ})} = 29.86cm.$

(iv) What is the total distance the ray has travelled from the object to the observer?

Answer

The total distance the ray has travelled from the object to the observer is $\frac{4}{\sin(15^{\circ})} + \frac{15}{\cos(15^{\circ})} = 30.98cm.$

(v) How far away from the observer is the image?

Answer

This is the same as the total distance from the object to the observer which is $30.98cm.$

(iii) What does the term 'lateral inversion' mean?

Answer

The apparent reversal of the image's left and right compared to the object.

7.3 Reflection - Concave Mirrors

(i) What is the name for the point to which rays parallel to the principal axis converge after being reflected from a concave mirror?

Answer

Focal point.

(ii) An object placed at the centre of curvature produces an image.

(i) What type of image is produced?

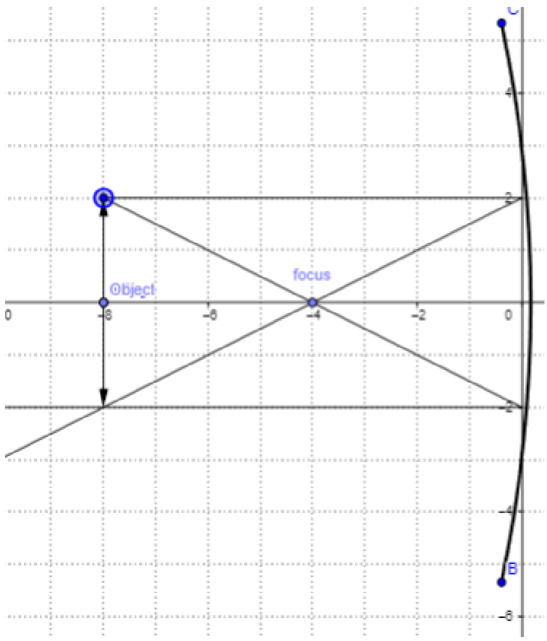
Answer

From the figiure it is easy to see that the image is real.

(ii) Where is the image produced?

Answer

The image is produced at the centre of curvature.



(iii) What is the orientation of the image?

Answer

Inverted.

(i) What is the size of the image?

Answer

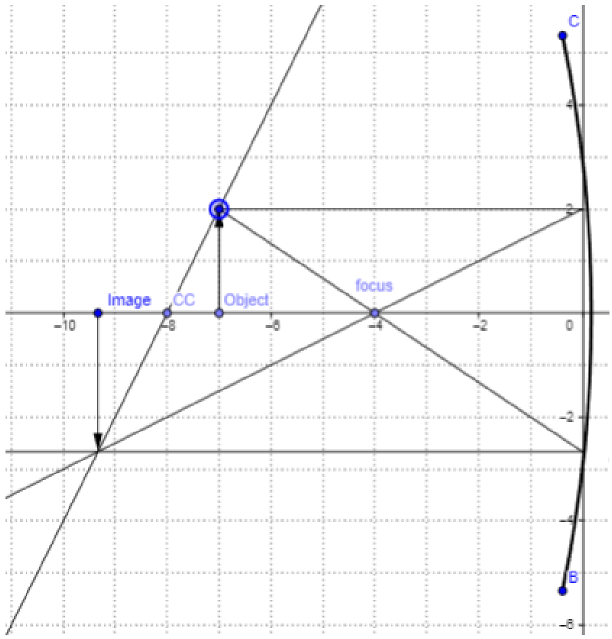
Same size as the object.

(iii) An object between C and F produces an image.

(i) What type of image is it?

Answer

From the figure below, it is easy to see that the image is real (same side as the object).



(ii) Where is the image?

Answer

The image is between C and infinity.

(iii) What is its orientation?

Answer

Inverted.

(iv) What is its size?

Answer

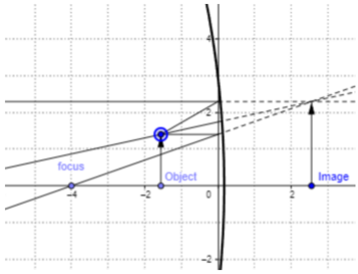
Enlarged.

(iv) An object placed between C and infinity produces an image.

(i) What type of image is it?

Answer

From the figure below, it is easy to see that the image is real (same side as the object).



(ii) Where is the image?

Answer

The image is between F and C.

(iii) What is its orientation?

Answer

Inverted.

(iv) What is its size?

Answer

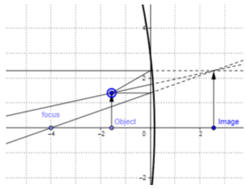
Diminished.

(v) An object placed between F and the mirror produces an image.

(i) What type of image is it?

Answer

From the figure below, it is easy to see that the image is virtual.



(ii) Where is the image?

Answer

The image is beyond the mirror.

(iii) What is its orientation?

Answer

Upright.

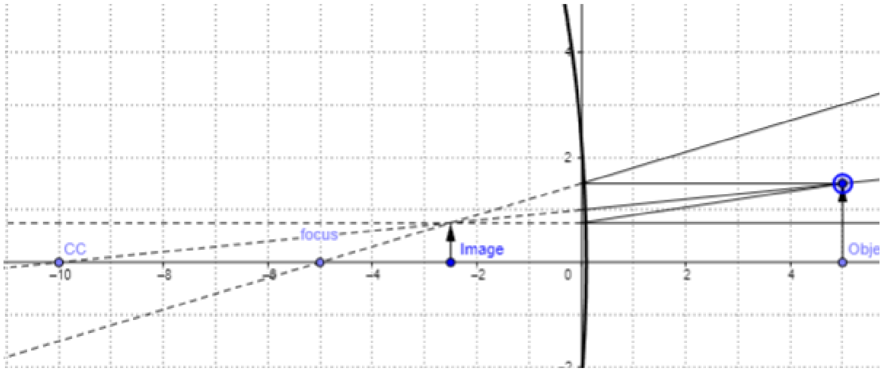
(iv) What is its size?

Answer

Enlarged.

7.4 Reflection - Convex Mirrors

(i) An object $1.5cm$ long is placed $5.0cm$ from a convex mirror. The radius of curvature of the mirror is $10.0cm$. Draw a scale ray diagram of the object and mirror.



(i) Use the scale diagram to measure the size of the image. Give your answer as a whole number of millimetres.

Answer

8 mm.

(ii) Use the scale diagram to measure the distance of the image from the mirror. Give your answer as a whole number of millimetres.

Answer

25 mm.

7.5 Refraction

Material	Speed of light (m/s)	Refractive Index
Air	3×10^8	1
Glass	1.9×10^8	$\frac{3}{1.9} \approx 1.6$
Water	2.3×10^8	$\frac{3}{2.3} \approx 1.3$
Diamond	1.2×10^8	$\frac{3}{1.2} \approx 2.5$
Turpentine	2×10^8	$\frac{3}{2} \approx 1.5$

(i) Does light never, not usually, usually or always bend towards the normal when going into a material with a higher refractive index with $i \neq 0$?

Answer

Light always travels slower in materials with a higher refractive index so will be bent towards the normal according to Snell's law.

(ii) Violet light is slower in glass than red light. All colours of light travel at the same speed in air. A narrow, white beam of light enters a glass block with $i = 30°$. Which colour bends the most on refracting as it enters the glass block?

Answer

The more the light is slowed, the more it bends so violet bends the most.

(iii) A wide beam of white light shines at an angle on a rectangular glass block, refracting on entry and on exit. Will the beam be parallel or diverging on leaving the block?

Answer The beam will be parallel. All the wavelengths of light are refracted by exactly the inverse amount that they underwent when the entered the block of glass when they leave the block of glass. As they entered parallel they leave parallel.

(iv) The wide beam of white light now shines on a glass block that is triangular when viewed from above. Will the beam be parallel or diverging on leaving the block?

Answer

The beam will be diverging. A triangular glass prism can be used to split white light into its constituent colours.

7.6 Total Internal Reflection

(i) The critical angle for light leaving a glass block into air is $42°$. The critical angle for light leaving water into air is $49°$. The critical angle for light leaving diamond into air is $24°$. The critical angle for light leaving cubic zirconia into air is $28°$. The critical angle for the glass/water boundary is $62°$.

Given the critical angle data, put the five materials (air, glass, water, diamond, and cubic zirconia) in order of increasing speed of light.

Answer

The slower light travels in a material, the higher its refractive index, and the smaller its critical angle at an air boundary. Therefore the order is:

(i) Diamond
(ii) Cubic Zirconia
(iii) Glass
(iv) Water
(v) Air

(ii) Complete the table, stating whether total internal reflection or refraction will occur

Answer

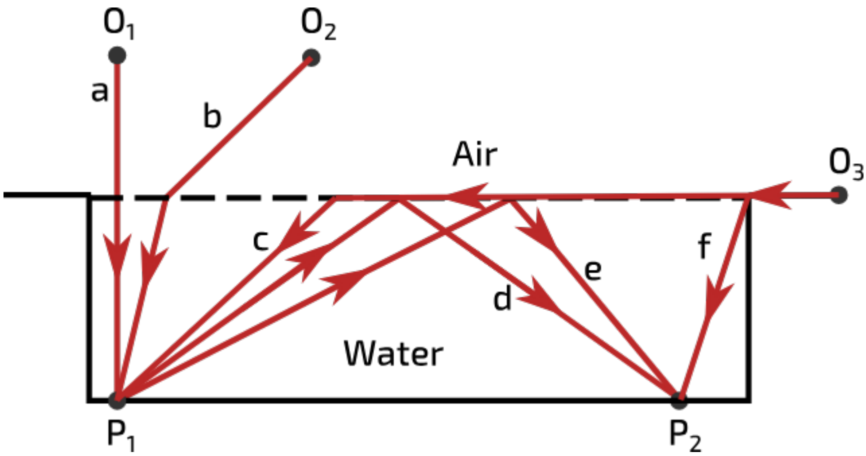
Light moving from	Light moving to	Angle of Incidence	What happens
Air	Glass	30°	Refraction
Glass	Air	30°	Refraction
Air	Glass	49°	Refraction
Glass	Air	49°	Total Internal Reflection
Water	Air	43°	Refraction
Glass	Water	70°	Total Internal Reflection
Water	Glass	82°	Refraction
Diamond	Air	24°	Refraction
Cubic Zirconia	Air	24°	Refraction

(iii) The critical angle for the glass/water boundary is 62°. For total internal reflection to occur, from which medium must light hit the boundary?

Answer

Glass.Total internal reflection can only occur when light arrives at a boundary from the medium with the slower speed of light. Glass has a slower speed of light than water.

(iv) The diagram shows a side view of a swimming pool. On the bottom are two people, P_1 and P_2, trying to see objects O_1, O_2 and O_3 outside the pool. The observers are also trying to see each other without looking directly at each other. Which of the rays shown are possibilities for observing?



Answer

Ray (a) is possible as a normal ray doesnt refract. Ray (b) is possible as a light ray bends towards the normal when moving from air to water. Ray (c) is possible when it is at the critical angle. Ray (d) is possible as it shows total internal reflection. Ray (d) is not possible as angle of incidence and angle of reflection are not the same.

(v) The critical angle for light leaving a glass block into air is 42°. The critical angle for light leaving diamond into air is 24°. The critical angle for light leaving cubic zirconia into air is 28°.

A jewel looks more sparkly the less light it allows to escape through the rear surface, and the more it is able to totally internally reflect light at the back.

Which is the best material for making a sparkly jewel - cubic zirconia, glass or diamond?

Answer

Diamond has the highest refractive index and therefore the smallest critical angle. As it has the smallest critical angle it will have the most internal reflections and fewest refractions.

7.7 Diffraction

(i) In the following table, label whether diffraction can be seen (Y) or not (N).

Answer

		Aperture size		
		10m	10cm	0.01mm
Wavelength	10m	-	Y	Y
	10cm	N	-	Y
	0.01mm	N	N	-

Diffraction depends on the wavelength and the size of the aperture. Waves spread more when the aperture is smaller than the wavelength.

(ii) In the following table, label whether there is an obvious shadow behind the obstacle (Y) or not (N).

Answer

		Obstacle size		
		50cm	25cm	10cm
Wavelength	50cm	-	N	N
	25cm	Y	-	N
	10cm	Y	Y	-

Diffraction depends on the wavelength and the size of the obstacle.Waves spread more when the aperture obstacle is smaller than the wavelength.

(iii) A young astronomer has a telescope with a $6.0cm$ diameter lens, and uses it to take pictures using visible light (wavelength =$500nm$). The main factor causing blurring in a good telescope is diffraction. If a professional astronomer wanted images just as precise using 30cm radio waves, what diameter of dish would be needed?

Answer

The ratio of aperture width to the wavelength need to be the same in both cases. Therefore, the diameter of the dish is $\frac{0.06\times0.3}{500\times10^{-9}} = 36km$.

(iv) A person with perfect eyesight can only read a message written using $1.0mm$ pixels if it is closer than $14m$ (with a pupil diameter of $7.0mm$)

because diffraction caused by the pupil blurs light from one pixel into another at greater distances.How far away could they read the same message if they used binoculars with 25mm diameter lens?

Answer

Let the distance in case of the binoculars be dm. For the diffraction angle to remain the same, we have $\frac{0.007}{14} = \frac{0.025}{d} \Rightarrow \mathbf{50}\boldsymbol{m}.$

(v) For satellite communications, radio and microwave transmission dishes need to be wide enough to prevent excess diffraction.Rank the following situations with the most parallel beam first.

(i) What rank is a $3.0cm$ microwave from a $21cm$ radius dish in terms of the diffraction with 1 being the most parallel beam?

(ii) What rank is a $15cm$ radar from a $3.0m$ radius dish in terms of the diffraction with 1 being the most parallel beam?

(iii) What rank is 500nm light from a 8.0μm blood cell in terms of the diffraction with 1 being the most parallel beam?

Answer

The amount the waves spread out depends on the wavelength divided by the width of the object (source or aperture). The higher the ratio, the more the diffraction.

For 1, $\frac{\lambda}{\text{wavelength}} = \frac{3}{21} = \frac{1}{7}$. For 2, $\frac{\lambda}{\text{wavelength}} = \frac{15}{300} = \frac{1}{20}$. For 3, $\frac{\lambda}{\text{wavelength}} = \frac{500\times10^{-9}}{8\times10^{-6}} = \frac{1}{16}$.

Therefore in case 1 the beam is least parallel, in case 2 the beam is more parallel than case 3.

7.8 Seismic Waves and Earthquakes

(i) The average speed of S waves in the mantle is 6.0km/s and the average speed of P waves in the mantle is 11km/s. Ignore the crust, and treat the mantle as being 2900km deep.

  (i) How much time would an S wave take to travel from a seismic event and return to the focus after having reflected once from the mantle-outer core boundary?

  Answer

  The total distance that the S wave would have to travel is $2 \times 2900 = 5800km$. The time taken is $\frac{5800}{6} = 967s$.

  (ii) How much earlier would the reflected P waves be detected than the S waves?

  The time taken to detect the P wave is $\frac{5800}{11} = 527s$. Therefore the P waves would be detected $967 - 527 = 440s$ earlier.

(ii) Explain how the location of an earthquake's focus can be worked out from the distance measurements made from three seismometers.

Answer

Spheres of radii of the measured distances can be drawn around each seismometer.Where the 3 spheres intersect will be the focus of the earthquake.

7.9 Refractive Index and Snell's Law

(i) Data:

refractive index of glass=1.50
refractive index of water=1.34
refractive index of air=1.00 speed of light in a vacuum=3.00×10^8m/s

Calculate the speed of light in the following materials.

(i) What is the speed of light in glass?

Answer

The speed of light in glass is $3.00 \times \frac{10^8}{1.5} = 2.0 \times 10^8$ m/s.

(ii) What is the speed of light in water?

Answer

The speed of light in water is $3.00 \times \frac{10^8}{1.34} = 2.23 \times 10^8$ m/s.

(ii) The speed of light in hydrogen disulphide is 1.59×10^8 m/s.Calculate the refractive index of hydrogen disulphide.

Answer

The refractive index of hydrogen sulphide is $3.00 \times \frac{10^8}{1.59} \times 10^8 = 1.89.$

(iii) A ray of light makes an angle of incidence of $40°$ with the normal between air and a liquid. The angle of refraction in the liquid is $28°$.Calculate the value of the refractive index of the liquid.

Answer

Let r_l be the refractive index of the liquid. Using Snell's law, we have $\frac{\sin(40^\circ)}{\sin(28^\circ)} = \frac{r_l}{1} \Rightarrow r_l = 1.37.$

(iv) The refractive index of a kind of glass for a certain wavelength of red light is 1.51. It is 1.55 for violet. A ray of white light is incident on a prism made of the glass at $30°$ to the normal.Calculate the angle between the red and violet rays in the glass.

Answer

Let r_V be the angle of refraction for violet light and let r_R be the angle of refraction for red light. Using Snell's law for violet light, we have $\frac{\sin(30^\circ)}{\sin(r_V)} = \frac{1.5}{1} \Rightarrow r_V = 19.34^\circ$. Using Snell's law for red light, we have $\frac{\sin(30^\circ)}{\sin(r_V)} = \frac{1.55}{1} \Rightarrow r_R = 18.82^\circ$. The angle between the red and violet rays is $19.34 - 18.82 = 0.52^\circ$.

(v) A ray of monochromatic light, travelling through air, makes an angle of $30°$ with the surface of a rectangular block of a certain type of transparent plastic. It makes an angle of $53°$ with the surface inside the plastic.What is the value of the plastic's refractive index?

Answer

The angle of incidence is $90^{\circ} - 30^{\circ} = 60^{\circ}$ and the angle of refraction is $90^{\circ} - 53^{\circ} = 37^{\circ}$. Let r_p be the refractive index of plastic. Therefore from Snell's law, we have $\frac{\sin(60^{\circ})}{\sin(37^{\circ})} = \frac{r_p}{1.0} \Rightarrow r_p = 1.44$.

(vi) Data:

refractive index of glass=1.50
refractive index of water=1.34
refractive index of air=1.00

A block of glass is sitting on the bottom of a tank full of water. Light enters the glass from the water with an incident angle $i = 40°$. What will the angle of refraction (r) in the glass be?

Answer

From Snell's law, we have $\frac{\sin(40^{\circ})}{\sin(r)} = \frac{1.5}{1.34} \Rightarrow r = 35^{\circ}$.

7.10 Calculating Critical Angles

(i) The critical angle for light leaving a particular type of glass into air is $38.4°$.

Answer

Let η be the refractive index of the glass. From Snell's law, we have $\frac{\sin(90^\circ)}{\sin(38.4^\circ)} = \frac{\eta}{1} \Rightarrow \eta = 1.6.$

(ii) Data:

refractive index of glass=1.50
refractive index of water=1.34

What is the critical angle for light passing from glass to water?

Answer

Let x be the critical angle From Snell's law, we have

$$\frac{1.50}{1.34} = \frac{\sin(90^\circ)}{\sin(x^\circ)} \Rightarrow x = 63.3^\circ. \qquad (7.1)$$

(iii) A thin ray of monochromatic light enters a block of pure ice at an angle of $42.0°$ to the normal from the air and the refracted angle in the ice is $30.7°$.Calculate the critical angle for ice.

Answer

From Snell's law, we have

$$\frac{\sin(42^\circ)}{\sin(37^\circ)} = \frac{\sin(90^\circ)}{\sin(x^\circ)} \Rightarrow x = 49.73^\circ. \qquad (7.2)$$

(iv) A tube of glass of refractive index 1.65 is surrounded by glass of refractive index 1.51. Calculate the critical angle for light travelling along the tube and incident on the boundary between the glasses.

Answer

Let x be the critical angle. From Snell's law, we have

$$\frac{1.65}{1.51} = \frac{\sin(90^\circ)}{\sin(x^\circ)} \Rightarrow x = 66.2^\circ. \qquad (7.3)$$

(v) In entering a transparent material from the air, the wavelength of a laser's light decreases from $600nm$ to $451nm$.Calculate the refractive index of the material.

Answer

Let η be the refractive index of the material.As the frequency of the light remains unchanged, the ratio of wavelenghts gives us the ratio of speeds. Therefore, we have $\frac{\eta}{1} = \frac{600}{451} \Rightarrow \eta = \mathbf{1.33.}$

7.11 Convex Lenses

(i) Work out the image distance and magnification for the following convex lenses, and state whether the image is real or virtual.

  (i) $f = 10cm$, object distance=$20cm$. What is the image distance?

  Answer

  We have $f = 10cm$ and $u = 20cm$.. Let v be the image distance. We have $\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{10} - \frac{1}{20} = \frac{1}{20} \Rightarrow v = 20cm$.

  (ii) What is the magnification of the image?

  Answer

  The magnification of the image is given by $\frac{v}{u} = \frac{20}{20} = 1$.

  (iii) Is the image real or virtual?

  Answer

  As v is positive, the image is real.

  (iv) When $P = 4.0D$, object distance =$10cm$. What is the image distance?

  Answer

  When $P = 4D$, $f = 0.25m$. The image distance is given by $v = \frac{f \cdot u}{u - f} = (.25 \times 0.1)(0.1 - 0.25) = -0.17m$.

  (v) What is the magnification of the image?

  Answer

  The magnification of the image is given by $\left|\frac{v}{u}\right| = \frac{0.17}{0.1} = 1.7$.

    (i) Is the image real or virtual?

  Answer

  As v is negative, the image is virtual.

(ii) Which lens has the greater optical power: a lens of focal length $10cm$ or one with a focal length of $50cm$?

Answer

When $f = 50cm, P = \frac{1}{0.5} = 2D$. When $f = 10cm, P = \frac{1}{0.1} = 10D$. Therefore the lens with focal length $10cm$ has greater power.

7.12 Concave Lenses

(i) Use a scale diagram or the formula to work out the image distance for each situation below. Assume that you have a $5.0cm$ concave lens ($f = -5.0cm$), and that the object height is $8.0cm$.

Answer

We have $\frac{1}{-5} = \frac{1}{20} + \frac{1}{v} \Rightarrow v = -4cm.$

(i) When object distance$= 10cm$, what is the image distance?

Answer

We have $\frac{1}{-5} = \frac{1}{10} + \frac{1}{v} \Rightarrow v = -3.33cm.$

7.13 Intensity and Radiation

(i) Calculate the intensity you would expect from a $1.0W$ torch bulb at a distance of $3.0m$.

Answer

The intensity is $\frac{1}{4\pi 3^2} = 0.0088\frac{W}{m^2}$.

(ii) What is the intensity of the light $7.0cm$ from a $100W$ bulb?

Answer

We imagine a sphere of radius $7cm$ enveloping the bulb. The surface area of the sphere is $4\pi(0.07^2)m^2$. We have $I = \frac{P}{A} = \frac{100}{4\pi(0.07)^2} = 1624\frac{W}{m^2}$.

(iii) What is the intensity of sunlight at the surface of the Earth?

Answer

If you hold a $100W$ bulb about $7.0cm$ from your eye, it looks as bright as the Sun so the intensity of the sun at the surface of the earth is $1624\frac{W}{m^2}$.

(iv) You want to make a solar power station giving an output of $2GW(2 \times 10^9 W)$. Use your answer to the previous question. Calculate the area of ground needed for solar cells if they are 100% efficient.

Answer

We know the power required and the intensit, therefore the area required is $2 \times \frac{10^9}{1624} = 1.23 \times 10^6 m^2$.

(v) Calculate the area of ground needed for solar cells if they are 20% efficient.

Answer

If the cells are 20% efficient, we need 5 times the area so the required area is $6.2 \times 10^6 m^2$.

8 Nuclear

8.1 Atomic Number and Nomenclature

(i) The following table gives the atomic number Z for various elements:

Element	**Z**
Hydrogen(H)	1
Helium(He)	2
Lithium,(Li)	3
Nitrogen(N)	7
Oxygen (O)	8
Uranium (U)	92

a. How many protons are there in a helium atom?

Answer

b.

b. How many electrons are there in a uranium atom?

Answer

cn.

c. How many protons are there in a lithium-7 atom?

Answer

c.

d. How many neutrons are there in a lithium-7 atom?

Answer

d.

(ii) State the number of protons, neutrons and electrons in $^{63}_{29}Cu^{+}$ ion.

a. State the number of protons.

Answer

ac.

b. State the number of neutrons.

Answer

63-29=34.

c. State the number of neutrons.

Answer

63-29=34.

d. State the number of electrons.

Answer

As the ion is positively charged, there should be one less electron than the number of protons. The number of electrons is $\mathbf{28}$.

(iii) Protons and neutrons are made of quarks. Up quarks (u) have a charge of $+\frac{2}{3}$ while down quarks (d) have charge $-\frac{1}{3}$. There are three quarks in a proton.

a. How many of them are up quarks?

Answer

As the proton is positively charged and there are three quarks in total. The number of up quarks is $\mathbf{2}$.

b. How many of them are down quarks?

Answer

The proton has three quarks in total of which two of them are up quarks. Therefore the number of down quarks is $\mathbf{1}$.

(iv) Protons and neutrons are made of quarks. What is a neutron made of?

Answer

As the neutron is electrically neutral, it has one up quark and two down quarks.

(v) During beta minus decay, a neutron turns into a proton. What happens in terms of quarks?

Answer

The proton has two up quarks and one down quark. The neutron has one up quark and two down quarks. Therefore during a beta minus decay, a down quark turns into an up quark.

(vi) During beta plus decay, a proton turns into a neutron. What happens in terms of quarks?

Answer

The proton has two up quarks and one down quark. The neutron has one up quark and two down quarks. Therefore during a beta plus decay, an up quark turns into a down quark.

8.2 Radioactive Decay

(i) Write an equation for the alpha decay of ${}^{238}_{92}U$.

Answer

${}^{238}_{92}U \to Th + {}^{4}_{2}\alpha.$

(ii) Write an equation for the beta minus decay of ${}^{14}_{6}C$ into N .

Answer

${}^{14}_{6}C \to N + {}^{0}_{-1}\beta.$

(iii) Write an equation for the beta plus decay of ${}^{14}_{8}O$ into N .

Answer

${}^{14}_{8}O \to N + {}^{0}_{+1}\beta.$

8.3 Half Life

(i) $^{14}_{6}C$ has a half life of 5700 years. A sample is 5700 years old and has an activity of 200 Bq.

a. What was the initial activity?

Answer

The half-life of the sample is 5700 years and the sample is 5700 years old. The current activity is $200Bq$ so the initial activity a half-life ago would have been $400Bq$.

b. What will the activity be 5700 years in the future?

Answer

After another half-life the activity will be half the current activity. Therefore it is $100Bq$.

(ii) A sample starts with 1×10^{16} nuclei of $^{3}_{1}H$, which has a half-life of 12 years.

a. How many $^{3}_{1}H$ nuclei will the sample contain after 12 years?

Answer

After 12 years i.e. one half-life, the sample will contain 5×10^{15} nuclei.

b. How many $^{3}_{1}H$ nuclei will the sample contain after 24 years?

Answer

After 24 years i.e. two half-lives, the sample will contain 2.5×10^{15} nuclei.

c. How many $^{3}_{1}H$ nuclei will the sample contain after 36 years?

Answer

After 36 years i.e. three half-lives, the sample will contain 1.25×10^{15} nuclei.

(iii) $^{13}_{7}N$, has a half-life of 10 mins.

a. If I start with 6000000 nuclei, how many will remain after 10 minutes?

Answer

After 10 minutes i.e. one half-life, the sample will contain 3000000 nuclei.

b. If the activity was 600 Bq initially, what will it be after 30 minutes ?

Answer

After three half-lives, the activity will be

c. If the activity was 24000 Bq initially, what will it be one hour later?

Answer

After 60 minutes i.e. six half-lives, the activity will be $\frac{24000}{2^6} = 375Bq.$

(iv) The half-life of ${}^{13}_{7}N$ is 10 mins. The initial activity of the sample is 100Bq.

Answer

After 5 minutes i.e. one half half-life, the activity will be $\frac{100}{2^{\frac{1}{2}}} \approx 71Bq.$

8.4 Fission - The Process

(i) When a nucleus of uranium-235 captures a neutron, fission takes place. One possible fission is:

$$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{95}_{36}Kr + ^{x}_{y}Ba + 3^{1}_{0}n \tag{8.1}$$

Calculate x and y.

Answer

Balancing the number of atomic number and the number of protons, we get the following equations:

$235 + 1 = 95 + x + 3$

$92 = 0 + 36 + y + 0$. Therefore the value of $x = 138$ and $y = 56$.

8.5 Energy from the Nucleus - Radioactivity and Fission

(i) Consider the reaction ${}_1^2H + {}_1^3H \to {}_2^4He + {}_0^1n.$

(i) Calculate the total mass of the reactants.

Answer

The total mass of the reactants is

$$3.3436 \times 10^{-27} + 5.0074 \times 10^{-27} = 8.351 \times 10^{-27} kg. \quad (8.1)$$

(ii) Calculate the total mass of the products.

Answer

The total mass of the products is

$$6.6447 \times 10^{-27} + 1.6749 \times 10^{-27} = 8.3196 \times 10^{-27} kg. \quad (8.2)$$

(iii) Calculate the energy lost.

Answer

The mass lost is given by $\Delta m = 0.0051 \times 10^{-27} kg.$ The energy lost is given by $\Delta mc^2 = 0.0314 \times 10^{-27} \times (3 \times 10^8) = 0.2826 \times 10^{-11} J.$

9 Gas

9.1 Boyle's Law

(i) a. What is $1.00cm^2$ in square metres?

Answer

$1cm = 0.01m$. Therefore $1cm^2 = 0.0001m^2$.

b. How much force does the atmosphere exert on a $1.00cm^2$ area?

Answer

$1Pa = \frac{1N}{m^2} = \frac{1N}{10000cm^2}$. Therefore atmospheric pressure is $1.01 \times 10^5 Pa = \frac{1.01 \times 10^5 N}{10000cm^2} = \frac{10.1N}{cm^2}$.

(ii) A barometer measures the pressure of the atmosphere in millibar (mbar), where $1.0mbar = 100Pa$. The surface area of the chamber in a barometer is $0.010m^2$, and the air pressure changes from $997mbar$ to $1013mbar$. What is the change in the force exerted by the air on the barometer's chamber?

Answer The change in pressure is $16mbar = 1600Pa = 1600\frac{N}{m^2}$. As the surface area of the chamber is $0.01m^2$, the change in force exerted on the chamber is $16N$.

(iii) Why does a gas exert a pressure on the walls of its container?

Answer In a gas, molecules move around rapidly.

They frequently collide with the wall. When this happens they bounce off with the same speed in a different direction.

This means that their velocity has changed and so their momentum must have changed, so there must have been a force on the molecule to cause this.

By Newton's Third Law there must also have been a force exerted on the wall.

(iv) $20cm^3$ of gas is at $100kPa$.

a. What will the pressure be if the gas is squeezed slowly down to $10cm^3$?

Answer

From Boyle's law, PV is constant so
$100kPa \times 20cm^3 = P \times 10cm^3 \Rightarrow P = 200kPa.$

b. What will the pressure be if the gas is allowed to expand slowly to $40cm^3$?

Answer

We have $100kPa \times 20cm^3 = P \times 40cm^3 \Rightarrow P = 50kPa.$

c. What will the volume be if the pressure is increased slowly to $1000kPa$?

Answer

We have $100kPa \times 20cm^3 = 1000kPa \times Vcm^3 \Rightarrow V = 2cm^3.$

d. Suppose the change in (a) was done really quickly. What would the effect be on the speed of the molecules?

Answer

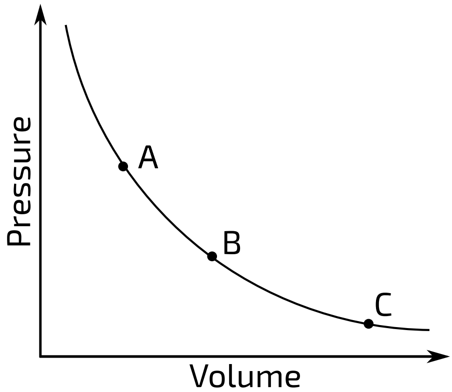
When the volume is decreased very quickly, the walls hit the molecules hard, giving them extra kinetic energy, so their speed increases.

e. Instead of a slow increase of pressure of the gas in (a), the change is made very quickly. Would the answer to question (a) now be lower or higher than the one you gave?

Answer

Higher.

(v) The co-ordinates of a point A on the line of a pressure-volume graph constructed for a fixed mass of gas at constant temperature are $(40, 30)$. Point $B(60, y)$ and point $C(x, 10)$ also lie on the line.



a. Calculate the value of x.

Answer

As PV is constant, we have $40 \times 30 = x \times 10 \Rightarrow x = 120.$

b. Calculate the value of y.

Answer

As PV is constant, we have $40 \times 30 = 60 \times y \Rightarrow y = 20.$

(vi) A certain car's suspension works by having a fixed mass of gas sealed inside a flexible capsule. Its pressure is usually $2.4 \times 10^5 Pa$ and its volume is 2.0 litres. On a bumpy road, at one point, the gas inside the capsule is compressed to 1.5 litres.

Answer

From Boyle's law, we have
$2.4 \times 10^5 Pa \times 2l = P \times 1.5l \Rightarrow P = 3.2 \times 10^5 Pa.$

(vii) A child lets a helium-filled balloon go and it floats up, higher and higher into the air, becoming larger and larger. When it was at ground level, its volume was $4000cm^3$ and the helium was at a pressure of $1.5 \times 10^5 Pa.$

Answer

From Boyle's law, we have
$1.5 \times 10^5 Pa \times 4000 cm^3 = P \times 6000 cm^3 \Rightarrow P = 10^5 Pa.$

(viii) In Boyle's Law, pressure and volume are inversely proportional ($p = \frac{k}{V}$, where k is a constant. What graph could you plot against p which would give a straight line?

Answer

As p is proportional to $\frac{1}{V}$, plotting p against $\frac{1}{V}$ will give a straight line. Plot p on the y-axis and $\frac{1}{V}$ on the x-axis.

(ix) Suppose the speed of each molecule doubled. What would happen to the pressure if the volume were fixed?

Answer

The temperature is proportional to the average kinetic energy of the molecules. When the speed of the molecules is doubled, the kinetic energy is quadrupled which means the temperature is quadrupled. The pressure is proportional to the temperature for a fixed volume of gas so the pressure is quadrupled.

(x) The average kinetic energy of molecules in air at $300K$ is $6.21 \times 10^{-21} J$. What is the average kinetic energy of molecules in air at $600K$?

Answer

As the temperature is proportional to the average kinetic energy of the molecules, the average kinetic energy of molecules in air at $600K$ is $12.42 \times 10^{-21} J$

(xi) The mass of a nitrogen molecule is 4.65×10^{-26} kg. The average value for (speed)^2 of a nitrogen molecule in air at $0^{\circ} C$.

Answer

As the temperature is proportional to the average kinetic energy of the molecules, the average kinetic energy of molecules in air at $0^{\circ} C = 273K$ is $5.7 \times 10^{-21} J$. We have
$\frac{1}{2} 4.65 \times 10^{-26} v^2 = 5.7 \times 10^{-21} = 2 \times 10^5 \frac{m^2}{s^2}$.

9.2 The Pressure Law

(i) I start with some gas at $30^{\circ}C$ at a pressure of $101kPa$ and heat it to $200^{\circ}C$. What will the new pressure be if I don't let the gas expand?

Answer

From the Pressure law, we have $\frac{101}{30+273} = \frac{P}{200+273} \Rightarrow P = 158kPa.$

(ii) I start with some gas at $-20^{\circ}C$, at a pressure of $101kPa$, and heat it until the pressure is $202kPa$ without letting it expand. What will the new temperature be?

Answer

From the Pressure law, we have $\frac{101}{273-20} = \frac{202}{273+T} \Rightarrow T =$

(iii) A cylinder of compressed gas is at a temperature of $23^{\circ}C$. It is cooled until it reaches a pressure of $2000kPa$. It has to be cooled to $90K$ before this happens.

Answer

(iv) If gas at atmospheric pressure $(101kPa)$ and at $300K$ is heated at constant volume to increase its pressure by 10%, what is the new temperature?

(v) What is the percentage decrease in pressure when air at $15^{\circ}C$ is cooled down to $-5.0^{\circ}C$ at constant volume?

(vi) A rigid container risks rupturing if the pressure of air within it rises above 230kPa. Assuming it initially contains air at $110kPa$ and $15^{\circ}C$, and is sealed, what is the maximum temperature to which the air can be heated without risk of rupture? Give your answer to the nearest degree.

(vii) What is the special name for the temperature of $-273^{\circ}C$?

Answer Absolute zero

9.3 Charles' Law

(i) I start with $20cm^3$ gas at 30°C and heat it to $200^{\circ}C$. What will the new volume be if I don't let the pressure build up?

Answer

Let the new volume be V. From Charles' law, we have
$\frac{20}{273+30} = \frac{V}{273+200} \Rightarrow V = 31.2cm^3$.

(ii) I start with $50cm^3$ gas at $-20^{\circ}C$ and heat it until the volume is $100cm^3$ without letting the pressure build up. What will the new temperature be?

Answer

Let the new temperature be $T^{\circ}C$. From Charles' law, we have
$\frac{50}{273-20} = \frac{100}{273+T} \Rightarrow T = 233^{\circ}C$.

(iii) You want to store 150litres of gas in a cylinder which only has room for 100litres. You can do this by reducing the temperature. The 150litres was measured at $15.0^{\circ}C$. How cold will you have to make it in order that it will fit in the cylinder at the same pressure?

Answer

Let the new temperature be $T^{\circ}C$. From Charles' law, we have
$\frac{150}{273+15} = \frac{100}{273+T} \Rightarrow T = -81^{\circ}C$.

(iv) A gas thermometer is made of a narrow cylinder closed at one end, with a fixed mass of gas inside, and a tight-fitting yet low-friction piston near the other end. The piston moves to ensure that the contained gas is always at atmospheric pressure. The cylinder contains helium gas, occupying a length of $134.6cm$ when at $22.4^{\circ}C$. How long is the gas column when the temperature is $-20.0^{\circ}C$?

Answer

As the thermometer is a cylinder, the volume of the gas is directly proportional to the length of the gas column. Let l be the length of the gas column at $-20^{\circ}C$.As the pressure is always atmospheric pressure from Charles's law we have $\frac{l}{134.6} = \frac{273-20}{273+22.4} \Rightarrow l = 115.2cm$.

9.4 General Gas Law

(i) I start with $5.0cm^3$ of gas at $30^{\circ}C$ at a pressure of $101kPa$ and heat it to $200^{\circ}C$. What will the new pressure be if I also compress it to $3.0cm^3$?

Answer

Let the new pressure of the gas be P. From the General Gas Law, we have $\frac{101\times5}{273+30}=\frac{P\times3}{273+200}\Rightarrow P=262.7kPa$.

(ii) The gas in a $0.020m^3$ cylinder is stored at 80bar (1bar=10^5 Pa). It starts at $290K$. The valve is opened until the gas pressure has equalized with the atmosphere (1.01bar). Assume that all of the gas is now at $280K$.How much volume does it take up?

Answer

Let the new volume of the gas be V. From the General Gas Law, we have $\frac{80\times0.02}{290}=\frac{1.01\times V}{280}\Rightarrow V=1.53m^3$.

(iii) I begin cycling with the air in my tyres at $270kPa$ and $285K$. Some time later, the air has warmed to $330K$, and the volume has increased by 3%. What is the new pressure?

Answer

Let the new pressure of the gas be P. From the General Gas Law, we have $\frac{270\times1}{285}=\frac{P\times1.03}{330}\Rightarrow P=303.52kPa$.

(iv) A gas bubble of volume $3.0cm^3$ forms at the bottom of a loch where the total pressure is 3.0atmospheres and the temperature is $4.0^{\circ}C$.What is its volume on reaching the surface where the water temperature is $13^{\circ}C$?

Answer

Let the volume at the surface be V. From the General Gas Law, we have $\frac{3\times3}{273+4}=\frac{V\times1}{273+13}\Rightarrow V=9.32cm^3$.

(v) The pressure in a flexible plastic flask is $1000kPa$ when its volume is $500cm^3$ and its temperature is $10^{\circ}C$. What would the pressure become if the gas volume was reduced to $400cm^3$ and it was heated to a temperature of $90^{\circ}C$?

Answer

Let the new pressure be P. From the General Gas Law, we have $\frac{1000\times500}{273+10}=\frac{P\times400}{273+90}\Rightarrow P=1603.35kPa$.

(vi) The gas in a spherical balloon is initially at $17^{\circ}C$. The temperature of the gas increases so that the pressure increases by 2% and the radius of the balloon increases by 4%. What is the new temperature of the gas (in celsius)?

Answer

If the initial volume is 1 and the radius is increased by 4%, the new volume is $(1.04)^3$. If the initial pressure is 1 and it is increased by 2%, the new pressure is 1.02. Let the new temperature be T. From the General Gas Law, we have $\frac{1.0\times1.0}{273+17} = \frac{1.02\times(1.04)^3}{273+T} \Rightarrow T = 59.7^{\circ}C$.